COHESIVE LAWS FOR DELAMINATION OF CFRP - EXPERIMENTS AND MODEL

Ulf Stigh
University of Skövde
Skövde, Sweden

ABSTRACT
In the present paper, we study delamination of a carbon fibre reinforced composite at a small length scale, i.e. without consideration of crack bridging. The study is performed within the framework of cohesive modelling. We propose methods based on the applications of the path independent J-integral to measure the cohesive law for delamination. With a DCB-specimen, the cohesive law corresponding to mode I loading is measured and with an ENF-specimen, the law corresponding to mode II loading is measured. These laws are used to calibrate a mixed-mode cohesive law. It is argued that the most important parameters of a cohesive law are the ability to provide the correct fracture energy and strength. The cohesive law is developed using a minimum of adjustable parameters to provide a transparent calibration process.

INTRODUCTION
Delamination of Carbon Fibre Reinforced Polymer composites (CFRP) is one of the major concerns in the design and use of composite structures. Delamination may start at unidentified defects originating from the production process or damages occurring in the use of the component. Two different mechanisms and length scales can be identified in the process of delamination. At the close proximity of a crack tip, a process region can be identified. With epoxy resins, the associated fracture energy is in the range of $10^{-2}$ N/m and the yield strength in the range of $10^1$ MPa. A simple estimate predicts the process zone to about $10^{-3}$ mm in size. That is large enough to imply interaction with the fibres. From an experimental point of view, this shows that the fracture properties should be measured in the relevant composite and one should not rely on bulk properties of the resin in predicting the strength of the composite. At the larger length scale and in the wake of a growing crack, crack bridging may occur. This process often contributes significantly and increases the total fracture energy to about $10^3$ N/m. The bridging stress is however small, in the range of $10^0$ MPa. That is, the process zone is very large, in the range of $10^1$ mm. Thus, the two fracture processes are associated with two very different length scales. In some applications, the enhancement of the strength due to crack bridging can be considered. However, in the aeronautic industry, no defects are allowed to grow during the use of a composite structure. Moreover, defects from the production are likely to lack bridging fibres. Therefore, if no defects from the production stage are allowed to grow during use, bridging cannot be considered in the design of aeronautical structures.

Cohesive modelling provides a simple method to introduce a process region in models of fracture. It is computationally attractive since it blends into the structure of finite element programmes for stress analysis. In front of a crack tip, a cohesive zone is introduced modelling the intricate fracture processes of the material. The zone can be viewed as two planar surfaces extending into the regular continuum model of the material. These surfaces are held together by tractions, assumed to be determined by the separation of the surfaces through a constitutive law, i.e. the cohesive law or the traction-separation relation. It should be stressed that in most applications, the cohesive law can be identified as modelling the joint actions of a number of fracture processes acting at the crack tip. These are often associated with a process region within a volume, that is, within the model, the actions of these processes in the volume are collapsed into a cohesive law acting on a cohesive surface without volume. This means that the cohesive law for a specific material is likely to be dependent on the constraints of the shape of the process region. In a laminated high-strength composite material, the goal is to fill the matrix with fibres. That is, even if the crack propagates in the weaker matrix, the fracture processes interacts with the fibres. A cohesive law for delamination is therefore likely to differ from the cohesive law for the pure matrix material.

Methods to measure cohesive laws have been developed during recent years. One class of such methods is based on the path-independence of Rice's J-integral, [1].

$$J = \int_C \left( U d\gamma - \mathbf{T} \cdot \frac{\partial \mathbf{u}}{\partial x} dC \right)$$  \hspace{1cm} (1)

where $U$, $\mathbf{T}$ and $\mathbf{u}$ denote the strain energy density, the traction and displacement vectors, respectively. A Cartesian coordinate system is used with the $x$ coordinate oriented along the
cohesive zone, cf. Fig. 1. It is noted that the material behaviour must be independent of $x$, i.e. $U$ is not allowed to be explicitly dependent of $x$. It is also noted that the material needs not be formally elastic as long as $U$ can be defined for the specific loading state. By choosing a path encircling the cohesive zone, $J$ is given by the area under

$$
J = \int_0^\delta \sigma(\delta) d\delta + \int_0^\tau(\tau) d\tau
$$

(1)

Where $\sigma$ and $\tau$ denote the normal $(y)$ and shear $(x)$ components of the traction vector, respectively; $w$ and $v$ denote the corresponding components of the separation vector. In some cases, $J$ can also be directly related to the applied load and deformation with relatively modest restrictions on the material behaviour, cf. e.g. [1]. In the present paper, we use two such specimens; the double cantilever beam (DCB) specimen and the end notched flexure (ENF) specimen. In [3], $J$ for the DCB-specimen is derived.

$$
J = \frac{2P\theta}{B}
$$

(2)

Where $P$, $\theta$ and $B$ are the applied force, the rotation of the loading point and the out-of-plane width of the specimen, respectively, cf. Fig. 2. The equation is valid for inelastic material behaviour as long as no unloading takes place before the crack starts to propagate. Unloading in the cohesive zone is however allowed for by using an inner integration path encircling the cohesive zone. It might be noted that the expression is independent of the crack length. This provides a possibility to use Eq. (2) to identify any change of the critical value of $J$ with crack propagation. It is also noted that Eq. (2) is valid for large deformations if $\theta$ is replaced with $\sin \theta$, [4].

When using specific structural theories, $\theta$ equals the rotation of the middle surface or neutral axis using the Kirchhoff plate theory and the Euler-Bernoulli beam theory, respectively. Using the Mindlin plate theory or the Timoshenko beam theory, $\theta$ should be taken as the average value of the rotations of the middle surface and the rotation of the cross section or the average of the rotation of the neutral axis and the rotation of the cross section, respectively [5].

In an experiment with the DCB-specimen, $P$, $\theta$ and $w$ are measured continuously. Using the path independence of the $J$-integral, Eqs. (1) and (2) are equated. After differentiation we arrive at

$$
\sigma(w) = \frac{d}{B \, dw} (P \theta)
$$

(3)

Thus, the cohesive law is derived from the experiment.

With the ENF-specimen, we correspondingly derive

$$
\tau(v) = \frac{d}{dv} \left[ \frac{18\alpha^2 P^2 a^2}{E_B H^3} - \frac{3\alpha P v}{E_B H} \right]
$$

(4)

where $v$, $\alpha$, $P$, $a$, $E_1$, $B$, and $H$ denote the shear deformation at the crack tip, the position of the load, the applied force, the crack length, Young’s modulus in the direction of the beam, the out-of-plane width, and the height of the beam, respectively.

Figure 3 illustrates the test specimen. In the derivation of Eq. (4), the specimen is assumed to be well represented by the Timoshenko beam theory, cf. [5.6]. It is also assumed that the cohesive zone is captured within the region between the crack tip and the loading point, i.e. within the length $b$ in Fig. 3. Moreover, the ENF-specimen is conditionally stable. That is, based on the assumptions above, a point shaped cohesive zone and constant fracture energy; the crack length must be larger than a critical value $a_c$, in order for the specimen to be stable under loading by controlled load point displacement, cf. [7]. In an experiment with the ENF-specimen, $P$ and $v$ are measured continuously during the experiment. Differentiation according to Eq. (4) gives the cohesive law in shear.

A fundamental problem in applying the methods above is to determine the appropriate length over which the separations $v$ and $w$ are to be measured. That is, according to the theoretical model, this length should be zero. However, considering that
the cohesive zone models the behaviour of the material within a volume, it is appropriate to measure over a length. A method to extract a cohesive law is developed in [8]. With this method, the cohesive law is adapted to experimentally measured data by optimization of the parameters to the experimental results. In the experiments, the expansion of the specimen at the crack tip is measured using the DCB-specimen. The evolution of \( J \) vs. the expansion is adapted. Figure 4 shows the result from one experiment. All data are normalized with respect to the mean critical value from all experiments in the series; these values are indicated by barred symbols. The material studied is a CFRP-laminate with all fibres in the longitudinal direction of the test specimens. The longitudinal and transversal elastic modulus are \( E_1 = 26G_{12} \) and \( E_2 = 1.9G_{12} \), respectively where \( G_{12} \) is the shear modulus. Poisson’s ratio is \( \nu_{12} = 0.3 \). Directions 1 and 2 correspond to the longitudinal and transversal direction in Figs. 2 and 3, respectively.

Since the expression for \( J \) is independent of \( w \), the fracture energy; i.e. the maximum value of \( J \), is considered as a given parameter in the optimization, cf. Eq. (2) and Fig. 5.

![Figure 4: Experimental curve is indicated by cross signs. Dashes indicate the optimal \( J(w) \) giving the curve indicated by the solid lines.](image)

The adapted cohesive law is a triangularly shaped curve determined by only three parameters, the peak stress \( \sigma_0 \), the corresponding separation, \( w_0 \), and the critical separation \( w_c \), cf. Fig. 5. These form the fracture energy \( J_c = \sigma_0 w_c/2 \), cf. Eq. (1).

![Figure 5: Left: Cohesive law used in the simulation model to back out the separation at the crack tip. Right: The parameters are constrained by the measured fracture energy.](image)

A digital image analysing system (Aramis) is used in the ENF-experiments. This system provides the displacement field in the crack tip region. Direct evaluation gives a good estimate of the shear, \( \nu \), at the crack tip. By use of Eq. (4), the cohesive law in shear results, cf. Fig. 6. In the present paper, a new cohesive law is presented. This law allows for a direct adaption to the cohesive laws measured in peel and shear. It also provides some flexibility in adaption to mixed mode loading.

### COHESIVE LAW

The starting point for the cohesive law is the measured properties in peel and shear. A trapezoidal non-dimensional cohesive law is first defined according to

\[
S(\lambda) = \begin{cases} 
\frac{\lambda}{\lambda_1} & \text{if } \lambda \leq \lambda_1 \\
1 & \text{if } \lambda_1 < \lambda \leq \lambda_2 \\
\frac{(1-\lambda)}{(1-\lambda_2)} & \text{if } \lambda_2 < \lambda \leq 1 \\
0 & \text{otherwise}
\end{cases}
\]  

(5)

where

\[
\lambda = \sqrt{\left(\frac{\nu}{\nu_c}\right)^2 + \left(\frac{w}{w_c}\right)^2}
\]  

(6)

is a non-dimensional measure of the separation of the cohesive surface where \( \lambda \geq 0 \) and with \( \lambda \geq 1 \) indicating fracture. That is, \( S = 0 \) for \( \lambda \geq 1 \). The parameters in Eq. (5) are related by \( 0 < \lambda_1 \leq \lambda_2 < 1 \). They are free to be fitted to the experimental results as explained below.

The mode mix is defined by

\[
\Psi = \frac{\nu^2}{\nu^2 + w^2}
\]  

(7)

Thus, pure mode I (peel) loading corresponds to \( \Psi = 0 \) and pure mode II (shear) loading to \( \Psi = 1 \) where \( 0 \leq \Psi \leq 1 \). The \( S(\lambda) \)-relation is now scaled to give
where \( f \) and \( g \) are two properly defined functions of the mode mix with the properties \( f(0) = 1 \) and \( g(1) = 1 \). With these properties \( \hat{\sigma} \) and \( \hat{\tau} \) are identified as the strengths in mode I and II, respectively. Thus, the model consists of six parameters: \( \hat{\sigma}, \hat{\tau}, w_c, v_c, \lambda_1 \) and \( \lambda_2 \), and two functions \( f(\Psi) \) and \( g(\Psi) \) to be adapted to the experimental results. With \( g(0) = 0 \), the shear stress is zero at pure mode I loading which is the only conceivable value. It is noted that by choosing \( f(1) \neq 0 \) a non-zero peel stress is achieved in pure shear. In shear loading, macroscopic crack growth is often associated with the formation of microscopic, slanted cracks in the process zone. In order for these to open, the process zone has to widen, i.e. giving \( w > 0 \). Thus, in Eq. (8), a negative \( f(1) \) is reasonable to achieve \( w = 0 \) in pure mode II loading, cf. [9] for a discussion related to shear loading of an adhesive layer. The fracture energy associated with the present cohesive law is given by

\[
J_c = \int_0^1 \sigma dv + \int_0^1 \tau dw
\]

Although \( \sigma \) and \( \tau \) are uniquely given by \( v \) and \( w \), \( J_c \) is dependent on the loading path. That is, no potential is associated with the present cohesive law. This can lead to unphysical behaviour. The remedy is to associate dissipative mechanisms to the law. This is developed in [5] and we note that the present cohesive law should be used with caution in cases where unloading is expected. The fracture energies in mode I and II are derived from Eqs. (5-9) to give

\[
\begin{align*}
J_{c1} &= \int_0^\infty \sigma \left| w_0 \right| dw = \frac{\hat{\sigma}w_0}{2} \left( 1 - \lambda_1 + \lambda_2 \right) \\
J_{c2} &= \int_0^\infty \tau \left| v_0 \right| dw = \frac{\hat{\tau}v_0}{2} \left( 1 - \lambda_1 + \lambda_2 \right)
\end{align*}
\]

It is noted that the following relation holds between the parameters of the model

\[
\frac{\hat{\sigma}w_0}{J_{c1}} = \frac{\hat{\tau}v_0}{J_{c2}}
\]

Thus, in the present model three of the most important parameters of a cohesive law are constrained by Eq. (11).

**INFLUENCES OF THE FUNCTIONS \( f(\Psi) \) AND \( g(\Psi) \)**

The choices of functions \( f(\Psi) \) and \( g(\Psi) \) sets the mixed-mode behaviour of the cohesive law. As discussed above, we are restricted to functions with the properties \( f(0) = 1 \), \( g(0) = 0 \) and \( g(1) = 1 \). Table 1 shows the present choices of functions used to study their influences. In the following evaluation, the following data are used: \( \lambda_1 = 0.1, \lambda_2 = 0.6, J_{c1}/J_{c2} = 7/3 \), and \( \hat{\tau}/\hat{\sigma} = 3/2 \). Figure 7 shows the influence of the different choices of the functions. All choices expect the Combined model provides a smaller fracture energy with increasing mode II loading at the left end of the graphs. That is, adding a small mode II loading to a mode I dominated loading, results in lower fracture energy. Usually, experiments show the reversed effect of adding some mode II loading. That is, the fracture energy increases, cf. e.g. [10].

**TABLE 1: Functions \( f(\Psi) \) and \( g(\Psi) \) used to study their influences on the mixed mode behaviour of the cohesive law**

<table>
<thead>
<tr>
<th>Model</th>
<th>( f(\Psi) )</th>
<th>( g(\Psi) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>( 1 - \Psi )</td>
<td>( \Psi )</td>
</tr>
<tr>
<td>Linear with compr.</td>
<td>( 1 - 3\Psi/2 )</td>
<td>( \Psi )</td>
</tr>
<tr>
<td>Quadratic</td>
<td>( 1 - \Psi^2 )</td>
<td>( \Psi^2 )</td>
</tr>
<tr>
<td>Quadratic alt.</td>
<td>( (1 - \Psi)^2 )</td>
<td>( \Psi^2 )</td>
</tr>
<tr>
<td>Combined</td>
<td>( 1 - \Psi^2 )</td>
<td>( \Psi )</td>
</tr>
</tbody>
</table>

The decreasing fracture energy occurs when the peel stress is assumed to decrease fast with the addition of a small amount of shear loading. That is, when \( f(\Psi) \) decreases fast with increasing \( \Psi \) at \( \Psi = 0 \). As observed, the two graphs corresponding to \( f(\Psi) = 1 - \Psi^2 \Rightarrow f'(\Psi) = 2\Psi \) which equals to zero at mode I loading, do have small or no decrease in \( J_c \) with the addition of a small amount of mode II loading.

Figure 8 shows the influence of mode-mix on the cohesive stresses using the Combined model. As discussed above, the peel stress decreases weakly with increasing mode II loading at a mode I dominated loading. That is, \( \sigma(w) \) with \( \Psi = 0.25 \) is only marginally smaller than for \( \Psi = 0 \). On the other hand, \( \tau(v) \) with \( \Psi = 0.75 \) is much smaller than for \( \Psi = 1 \). That is, a small amount of mode I loading in a shear dominated loading case influences the shear stress significantly.

**ADAPTION TO EXPERIMENTAL RESULTS**

In choosing parameters to fit the experimental results, some features of the cohesive law are considered more important than other. For instance, if the crack length is long compared to the size of the process zone, the fracture energy is the most...
important property of the cohesive law. To this end, we first chose $J_{Ic}$ and $J_{IIc}$. For shorter cracks, the influence of the strengths $\hat{\sigma}$ and $\hat{\tau}$ makes themselves noticed. Thus, these parameters are chosen to fit the experimental data.

![Figure 8: Effects of mode-mix on cohesive stresses for the Combined model in Table 1.](image)

Considering Eq. (11) and the actual shape of the cohesive law, e.g. Fig. 6, we suggest considering the raw data from the experiments and chose suitable values of $v_c$ and $w_c$. The parameter $\lambda_1$ reflects the elastic property of the cohesive law. If the cohesive law is assumed to model the splitting of the continuum in two parts, $\lambda_1$ should be set to as small value as possible. However, if we assume that the cohesive law reflects the behaviour of a material volume, $\lambda_1$ should be set to give a good estimate of the stiffness of the material volume. In mode II, the stiffness is estimated to about $\dot{\tau}/\lambda_1 v_c \sim 10^{12}$ N/m$^4$ corresponding to an about $10^{-4}$ mm thick process region.

Considering that the length of the process zone is estimated to about $10^{-1}$ mm, the estimates indicates a long a thin process region. Thus, some support is given to the use of a surface model of the process region. With these data we get $\lambda_1 = 0.38$. Equations (10) now provide two identical expressions for $\lambda_2 - \lambda_4$. With the present data, we get $\lambda_2 = \lambda_4$. With this, all parameters of the model are determined. The properties of the cohesive model in mode I and II loading are by this fully determined. The functions $f(\Psi)$ and $g(\Psi)$ remains to be chosen. These functions determine the properties in mixed mode loading and future studies will provide more insight into these properties.

**DISCUSSION**

With cohesive models, the intricate fracture processes acting at a crack tip are modelled with a cohesive law. With a few exceptions, these fracture processes occupy a material volume. This is reflected in cohesive stresses and corresponding fracture energies of the order of $10^7$ Pa and $10^3$ J/m$^2$, respectively. That is, about two orders of magnitude smaller cohesive stresses and three orders of magnitude larger fracture energies than expected from atomistic estimates. In the present application of delamination of a CFRP, no material volume is easily identifiable. As a result of the present procedure, a material volume is identified.

As shown here, the size of the process zone is small. This indicates that linear elastic fracture mechanics should be adequate for many applications. However, there is a growing demand for understanding the effects of minor defects. With defects of similar size to the process region, a model of the process region is necessary. The present cohesive model is one possibility. A cohesive model also allows for prediction of initiation of cracks in a structure without pre-cracks. Some promising attempts have been made to predict initiation of delamination without pre-cracks.

**CONCLUSIONS**

Cohesive models provide convenient methods to simulate delamination of CFRP. The models are computationally attractive. A number of methods have been developed to deduce or measure the cohesive law during the past twenty years. Of these methods, the ones based on the path independent $J$-integral are especially attractive, principally those that do not rest on too specific assumptions on the behaviour of the material of the test specimen.

Here, methods to measure the peel and shear properties are presented. A cohesive law is developed that allows for a direct identification of the parameters of the model in terms of the measured properties. For a complete description, data for mixed-mode loading are needed. The mixed-mode behaviour of the cohesive law is governed by the two functions $f(\Psi)$ and $g(\Psi)$ with the properties $f(0) = 1$ and $g(1) = 1$. It is argued that models with $f'(0) = 0$ are more in line with experimental results.

**NOMENCLATURE**

- $a$ Crack length [m]
- $\alpha$ Geometrical position [-]
- $B$ Out of plane thickness of specimen [m]
- $C$ Integration path [m]
- $\Delta$ Load point displacement [m]
- $E_i$ Young’s modulus in the fibre direction [N/m$^2$]
- $H$ Height of specimen [m]
- $J$ Energy release rate, $J$-integral [N/m]
- $J_c$ Fracture energy [N/m]
- $J_{Ic}, J_{IIc}$ Fracture energies in mode I and II [N/m]
- $l, L$ Length of specimen [m]
- $\lambda$ Non-dimensional separation [-]
- $\lambda_1, \lambda_2$ $\lambda$ at breaking points [-]
- $P$ Load [N]
- $\Psi$ Mode-mix [-]
- $S$ Non-dimensional stress [-]
- $\sigma, \tau$ Peel and shear cohesive stresses [N/m$^2$]
- $\hat{\sigma}, \hat{\tau}$ Strengths in pure peel and shear loading [N/m$^2$]
- $T$ Traction vector [N/m$^2$]
- $\theta$ Rotation of loading point [-]
- $u$ Displacement vector [m]
\( U \) Strain energy density \([\text{N/m}^2]\)

\( \nu \) Shear deformation \([\text{m}]\)

\( \nu_c \) Critical shear deformation \([\text{m}]\)

\( w \) Peel deformation \([\text{m}]\)

\( w_c \) Critical peel deformation \([\text{m}]\)

ACKNOWLEDGMENTS

This study is partly financed through the NFFP Projects KEKS and EFFEKT. The support is gratefully acknowledged.

REFERENCES


