Impact Simulation of Adhesively Joined Structures

THOMAS CARLBERGER

Department of Applied Mechanics
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden, 2006
THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

in

SOLID MECHANICS

IMPACT SIMULATION OF ADHESIVELY JOINED STRUCTURES

by

THOMAS CARLBERGER

Department of Applied Mechanics
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden, 2006
IMPACT SIMULATION OF ADHESIVELY JOINED STRUCTURES

THOMAS CARLBERGER

© THOMAS CARLBERGER, 2006

THESIS FOR LICENTIATE OF ENGINEERING no 2006:11
ISSN 1652-8565

Department of Applied Mechanics
Chalmers University of Technology
SE-412 96 Göteborg
Sweden
Telephone +46 (0)31 772 1000
Impact Simulation of Adhesively Joined Structures
THOMAS CARLBERGER
SAAB Automobile AB, SWEDEN

Abstract

The development of competitive and crashworthy automotive car bodies has reached so far that the manufacturers no longer rely on mono-material steel structures. Improved strength/weight performance may be achieved by using optimal material in each part of the car structure, leading to bi-material joints and ruling out spot welding; the common joining method during the past half century of automotive history. Adhesive bonding is an attractive joining method, not only capable of managing dissimilar materials, but also capable of improving stiffness and strength in mono-material structures. Moreover, the joints do not need additional sealing and there may be cost savings using adhesive bonding. Impact simulation of car structures is mostly performed using an explicit FE-method. This method has an inherent stability criterion: the time step used may not exceed the stable time step, or critical time step, $\Delta t_c$. In this thesis, a simplified cohesive zone model is studied. It is implemented into an explicit FE-code and compared to a closed form solution. The FE-solutions agree with the closed form solutions. It is found that the evolution of damage in the adhesive layer may stop under certain conditions that are likely to occur in a real structure. It is shown that an explicit FE-analysis with a “large” time step is more prone to give immediate rupture. Thus, the method is conservative. An interphase element formulation is derived for a 2D-adhesive joint model, joining beam adherends. It is shown that the mass matrix of the interphase element gives a small contribution to the mass matrix of the structure. However, this contribution is positive for the numerical stability of the explicit FE-method and it is recommended to keep this matrix in the analysis. Moreover, it is concluded that the contribution of material damping of the adhesive layer can be neglected as compared to the effects of plasticity of the adherends. The interphase element formulation is used to analyse the Double Cantilever Beam specimen. The results are compared to an alternative model using continuum elements. The comparison shows substantially faster convergence and shorter execution time for the interphase formulation. A rough estimate indicates fifteen times shorter execution time using the interphase elements in a realistic structure.

Keywords: Adhesive joining; Dynamic fracture; Cohesive zone; Interphase Element; Bi-material joining
Preface
The work presented in this thesis has been carried out during October 2003 until May 2006 at the University of Skövde in collaboration with Chalmers University of Technology. The work is a part of a co-operation program funded by the Swedish Consortium for Crashworthiness and SAAB Automobile AB.

I would like to express my sincere appreciation towards Prof. Ulf Stigh for his generous support and friendly way. I would also like to thank all my colleagues at Saab Automobile AB, the University of Skövde and Chalmers University of Technology for all discussions, help and laughs, making it possible for me to accomplish this task.

Finally, I would like to thank my wife Reem and my wonderful children Andreas, Caroline and Isabelle, who are the sunshine of my life.

Sjuntorp, May 2006
Thomas Carlberger
**Contents**

Abstract .......................................................................................................................i  
Preface ........................................................................................................................ii  
Contents ....................................................................................................................iii  
Introduction ................................................................................................................1  
Review of dynamic properties of adhesive joints and delamination .........................3  
Cohesive zone model .................................................................................................5  
Interphase element .....................................................................................................7  
Strain rate dependence ...............................................................................................8  
Practical test specimen ...............................................................................................9  
Conclusions ................................................................................................................9  
Future work ..............................................................................................................10  
Acknowledgements ..................................................................................................10  
References ................................................................................................................11

**Appended papers**

Paper A: Dynamic fracture of adhesive joints using explicit FE-code and the adhesive layer theory

Paper B: Explicit FE-formulation of interphase elements for adhesive joints
Introduction

Automotive manufacturers are facing increasing requirements regarding crash performance and weight saving. Optimal material selection in each part of the structure leads to bi-material joints. Adhesive joining may prove a welcome possibility by allowing dissimilar materials to be joined in a strong and cost efficient manner, and also improving strength/weight performance in single material structures. The engineering tool for impact simulation of car structures is explicit FE modelling. This method has an inherent stability criterion: the time step used may not exceed the stable time step, or critical time step, $\Delta t_c$, governed by the smallest element size, $l_{\text{min}}$, and the elastic wave propagation speed of the material, $c$, and estimated by

$$\Delta t_c = \frac{l_{\text{min}}}{c}.$$  \hspace{1cm} (1)

This time step limit, also called the Courant limit (or CFL-limit), cf. e.g. Belytschko (2000). It is likewise an issue when modelling spot welds. Typically, a time step of 1 µs is used, which corresponds to a minimum adhesive element length of about 1.2 mm. At a first glance, this seems not to be a severe problem, since modelling the adhesive joint between two shell structures by the use of continuum elements, would imply an adhesive thickness spanning the distance between the two mid planes of the shells in addition to the adhesive thickness, see Fig. 1(b).

![Figure 1. Cross sections of (a) spot weld joint model and (b) ad hoc adhesive joint model.](image)

For typical sheet thickness 0.8 mm and adhesive thickness 0.2 mm, this sums up to 1 mm, but as earlier mentioned, we have a minimum length of 1.2 mm. A common method to circumvent the Courant limit, often used when modelling spot welds, is the use of mass scaling. An artificially increased mass will reduce the wave speed and keep the time step at the desired size. This method adds excess inertia to the model and will give a more or less erroneous result in the vicinity of the added mass. In engineering practice, Young’s modulus is often artificially decreased to prevent the time step from decreasing, without the negative effect of added inertia (DuBois, 2005). However, also this method has draw-backs.
Moreover, the simplistic method to model the adhesive layer as indicated in Fig. 1b provides an erroneous evaluation of the deformation of the adhesive layer. If the adherends rotate and translate, the adhesive layer suffers shear deformation as indicated in Fig. 2; the ad hoc model gets a too small shear deformation. The *ad hoc* model of the adhesive as continuum between the shell mid planes gives the shear $\gamma = \frac{\delta_s}{(H_1/2 + H_2/2 + h)}$, while the correct shear is $\gamma = \frac{\delta_s}{h}$. To account for the rotation of the adherends, we will have to connect the continuum element to the shell elements by some rigid connections.

In this case, the adhesive layer thickness $h$ and the Courant limit cause a real problem. Modelling the adhesive with solid continuum elements would render a significantly shorter time step than otherwise required by the structure. Two alternatives to model the adhesive layer are worth scrutinising: the first is a continuum model and the second is the interphase element formulation. In paper B these models are developed, implemented and compared. Both these approaches are based on the adhesive layer theory, cf. Klarbring (1991). According to an asymptotic analysis, two deformation modes are shown to dominate the behaviour of an adhesive layer. These are denoted peel and shear deformation, respectively.

In the explicit FE-method the execution time, or run-time, $T_r$, is inversely proportional to the time step, $\Delta t$,

$$T_r \approx kn_{\text{DOF}} \frac{1}{\Delta t},$$

where $k$ is a constant and $n_{\text{DOF}}$ is the number of degrees of freedom (DOF) in the model. Effort should be made to model the adhesive in the most efficient way keeping the number of degrees of freedom, $n_{\text{DOF}}$, low, while keeping the time step, $\Delta t$, as large as required by the rest of the structure, not forgetting the important issue of the accuracy of prediction of adhesive ultimate strength.

To be able to perform reliable crash simulations, there has to be an understanding of the adhesive joint properties under impact conditions. Practical tests have to be
performed and evaluated in order to determine the relevant properties of the constitutive relations used for simulation of impact. For practical use in the automotive industry, it is preferable to have a simple test to perform with each new adhesive and adherend material combination to determine the parameters for use in the constitutive models for simulation. There have been many papers written on the subject of static fracture of adhesive joints, but relatively few on dynamic fracture. Adhesive test specimens are quite similar to composite test specimens, which are common research subjects. These are therefore included to some extent in the present work.

**Review of dynamic properties of adhesive joints and delamination**

This is a short review of selected research work on dynamic fracture of adhesives and composites.

Guo and Sun (1998) studied dynamic mode-I delamination of a carbon/epoxy composite in a modified DCB–specimen. They use a conductive circuit placed along the path of the propagating crack to measure the crack length. In this experimental setup Guo and Sun noted that the critical dynamic energy release rate is nearly constant and very close to the static fracture toughness of the composite, indicating low strain rate sensitivity for the composite. In the numerical simulations they show that the strain energy release rate at the straight crack front is not uniform and dips towards the free edges.

Maheri and Adams (2002) use the thick adherend shear test (TAST) to determine the dynamic shear modulus of structural adhesives. The basic principle behind this test is that the adherend is stiffer than the relatively compliant adhesive. Thus, the authors assume that the shear deformation is uniform in the adhesive layer. The strain measurements can then be substituted by displacement measurements of the adherends. In this application Maheri and Adams investigated the dynamic shear modulus under vibration load. With steel adherends, the tests showed that the compliance of the adherends is to be considered for cases $G/h > 1$ GPa/mm, where $G$ is the shear modulus of the adhesive and $h$ is the thickness of the adhesive layer. For many structural adhesives used in the automotive industry $G/h$ is considerably larger than 1 GPa/mm.

Kusaka et al. (1998) investigate the rate dependence of the mode-I fracture toughness of carbon fibre epoxy laminates. They perform experiments with the wedge-insert-fracture method using a split Hopkinson pressure bar to impact the wedge. A DCB-specimen is used for quasi-static fracture tests. They find essentially two levels of fracture toughness, one for fast experiments and one for slow experiments with a transition region. Crack growth behaviour changed from unstable in the lower and transition regions, to stable in the higher region. The authors interpret this as an effect of differences in static and dynamic fracture toughness.

Tsai et al. (2001) study the mode-II-dominated delamination of carbon fibre composites. Both energy balance and modified crack closure methods are used to evaluate the experiments. To produce high crack propagation speeds, a brittle adhesive film strip is placed in front of the prefabricated crack tip. Through the strips...
relatively high strength, elastic energy is stored in the specimen. At fracture of the strip, the energy is released instantly resulting in high crack velocity. The setup produce crack speeds well above 1000 m/s. A finite element simulation is performed and used to evaluate the experiments.

Kihara et al. (2003) conduct impact shear tests of adhesive layers using a split Hopkinson bar together with strain gauges to monitor shear strain waves in the adherends. The adherends are thick (TAST). The strain in the adhesive is not measured directly. An FE model is in this case used to calculate the stresses in the adhesive layer by comparing strain waves in the adherends calculated by FEM and measured in the test setup. Their conclusions are that this equipment could be used to measure the average stress in an adhesive layer. Fracture at low load is caused by peel stress; fracture at high load is due to a combination of shear and compressive peel stress. The authors conclude that the shear strength of an adhesive layer subjected to impact could be evaluated by this experimental equipment.

Blackman et al. (2003) suggest a two-parameter criterion for fracture instead of the conventional one-parameter criterion, governed by the critical energy release rate solely. The authors use three different types of specimens in a study of the two-parameter criterion. The types are (a) a DCB, (b) a tapered double cantilever beam (TDCB) and (c) the 90° peel test. The second parameter is suggested to be a critical limiting value of stress, \( \sigma_{\text{max}} \). A part of the paper is dedicated to determine \( \sigma_{\text{max}} \). Cohesive zone modelling (CZM), finite element analysis (FEA) and experiments are used to evaluate \( \sigma_{\text{max}} \). Tests show that it is not possible to fix the value of \( \sigma_{\text{max}} \) with any greater degree of certainty by simply comparing the CZM/FEA and experimental data. Interestingly, the value of \( \sigma_{\text{max}} \) is approximately the same as the ultimate tensile yield stress of the adhesive layer. The authors fail to use the CZM/FEA approach with a specified \( \sigma_{\text{max}} \) larger than 50 MPa due to numerical instabilities. If lower values of \( \sigma_{\text{max}} \) are specified, both the stiffness and the resulting load per unit width for crack growth decrease.

Blackman et al. (2000) use the impact wedge-peel test (IWP) to measure cleavage fracture of structural adhesives at a relatively high test-rate. The speed of the wedge is 2 to 3 m/s. The main objective is to evaluate the performance of a range of structural adhesives when used to bond steel or aluminium substrates. A servo-hydraulic machine together with a high-speed camera is the essential testing equipment used. Both stable and unstable crack growth is observed, with cohesive crack propagation as the failure mode through the adhesive layer. At the moment this paper was prepared a standard test procedure, ISO 11343 was developed. In this standard, the wedge driving force is measured as a function of time. This force/unit width is the main result of the experiment. A critical review of the proposed standard is performed. A comparison of the fracture energy evaluated with the IWP and TDCB-tests is performed. The authors find the fracture energy to be ten to fifteen times larger when evaluated with the IWP-test. During the tests, the adherends deform plastically, which explains this phenomenon. Finally, it is shown that a finite-element analysis of the IWP test geometry may predict the failure behavior successfully.

Borg et al. (2004) perform delamination simulations of the DCB, ENF and MMB (Mixed Mode Bending) specimens using a cohesive zone model to connect shell elements. They report results in agreement with experimental results. These
simulations are quasistatic. Their shell-based models account for the thickness offset and rotations of the shell adherends.

Generally, in these papers the strain rate is not measured in the adhesive. Instead the speed of the crack tip is measured. These two parameters are related but not equivalent. If strain rate dependence is to be incorporated into a material model of the adhesive and determined from experiments, it is necessary to determine the strain rate in the experiment. In paper A, Carlberger and Stigh (2006) show the multiplying effect between adherend strain rate and adhesive strain rate for the simple butt joint.

**Cohesive zone model**

Cohesive zone models have proven useful in simulating fracture, cf. e.g. DeBorst et al. (2006). The cohesive zone influences the critical time step of an explicit FE-analysis. This critical time step is given by $2/\omega_{\text{max}}$, which is often estimated according to the Courant limit Eq. (1). To calculate the highest eigenfrequencies of the structure, we will have to establish the structural stiffness matrix. This is not done in commercial explicit FE-codes, due to the large memory requirements associated with this task. Instead, Courant’s limit, Eq. (1), is used to decide the time step. The time step is subsequently reduced by special algorithms dedicated to estimate the effects of e.g. contact stiffness. This estimate of the highest eigenfrequency corresponds to the largest eigenfrequency of all elements in the mesh evaluated as if the elements are free. A simple example is given in Fig. 3a.

![Figure 3. Simplified structures. (a) with cohesive zone of stiffness $K$, (b) rigidly connected without cohesive zone, and (c) free bar.](image)

A primitive method to connect the shells of an adhesively bonded structure is simply to connect the nodes of the two bonded shells. This method can be argued for considering the large bending flexibility of the shells as compared to the flexibility of the adhesive layer. Figure 3b indicates this procedure. More accurately, the adhesive is given flexibility; Fig. 3c gives a simple example. The maximum eigenfrequencies for the models in Fig. 3 are
\[
\omega_{\text{max}} = \begin{cases} \sqrt{\frac{k}{m}} & \text{(a)} \\ \sqrt{\frac{2k}{m}} & \text{(b)} \\ \sqrt{\frac{k}{m} \left[ 2 + \frac{K}{k} + \sqrt{4 + \left(\frac{K}{k}\right)^2} \right]} & \text{(c)} \end{cases}
\]

where \( K \) is the stiffness of the cohesive zone. The mass of the bar is \( m \). The stiffness of the bar, \( k \), is given by Young’s modulus, \( E \), the cross-sectional area, \( A \), and the length, \( l \), of the bar,

\[
k = \frac{EA}{l}. \tag{4}
\]

The mass of the bar, \( m = \rho Al \) where \( \rho \) is the density of the bar. With typical data for a component in a car: \( E = 210 \text{ GPa}, l = 5 \text{ mm}, A = 1 \text{ cm}^2, \rho = 7800 \text{ kg/m}^3 \) and cohesive zone stiffness \( K = 2 \text{ GN/m} \). The results are presented in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>free bar (Fig. 3a)</th>
<th>rigidly connected (Fig. 3b)</th>
<th>cohesive zone (Fig. 3c)</th>
<th>Eq. (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta t_c/\mu s )</td>
<td>0.96</td>
<td>1.36</td>
<td>0.90</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Table 1. Critical time steps for simple structures compared to the estimated limit, Eq. (1).

As evident in Table 1, the cohesive stiffness affects the critical time step of the bar. When comparing the model with a cohesive zone to the rigidly connected bar, the time step has to be substantially reduced. However, as compared to the Courant limit used, Eq. (1) and the free bar the reduction is marginal (6%).

Now, compare the critical time step for the structure in Fig. 3c with the critical time step for an identical structure, but with the cohesive zone exchanged for a typical adhesive continuum element of element length \( l = 0.2 \text{ mm} \) and the wave speed \( c = 1200 \text{ m/s} \) (epoxy). The continuum structure renders a critical time step size = 0.167 \( \mu s \) or almost one sixth of the size of the time step using the cohesive zone model. Thus, the cohesive zone seems to be a possible way of keeping the time step size large.

In paper A, a simple model containing a cohesive zone is developed. The model is given typical dimensions for automotive structures. To study how the cohesive zone influences the response of a structure subjected to impact, a simplified impact bar model is suggested, cf. Fig. 4.
Figure 4. Simplified impact bar model. The left end is connected to a rigid wall by means of a cohesive force $F_{\text{coh}}$, and the right end of the bar is subjected to a step load $F_{\text{load}}$ of magnitude $\sigma_0 A$.

The cohesive zone is given different typical characteristics as linear elasticity, linear softening with damage and the combination thereof. One aspect of the comparison is dedicated to the ability of the FE-method to render the correct stress behaviour during an impact situation; another is dedicated to the effect of the cohesive zone properties on the structural behaviour. Furthermore, we investigate the effects of a damage parameter on the solution. It is noticed that the time step size influences the fracture properties of the model, in such a manner that it is conservative. That is, with a large time step fracture is more likely to be predicted. The reason is that the FE-code evaluates the stress in the cohesive zone using the deformation from the previous time step. This results in an artificially too small fracture energy when using a large time step. Furthermore, the size and wave speed of the bar are found to influence the characteristics of the solution. The adhesive joint may either stay in the elastic region, or it may get partially damaged during several stress waves and finally fracture or the evolution of damage may stop. Finally, the structure may rupture during the first stress wave.

During the event of an automotive crash, the first contact between the car structure and the opposing object sends a stress wave propagating at the wave speed, $c$, through the car structure. In less than 1 ms, the complete car structure will be passed by the stress wave. This time can be compared to the duration of a typical crash event, about 100 ms. The adhesive joints will be subjected to these stress waves as they propagate back and forth through the structure. It is plausible to assume that the adhesive joints will be partially damaged at a certain point of time and that the following stress waves will accumulate damage, only occasionally leading to rupture. The model of the adhesive joints will thus have to be able to handle multiple loading and unloading, with and without damage, in a correct manner. A part of paper A is dedicated to this study and it is shown that the described situation is handled properly.

**Interphase element**

From a modelling point of view, there is an advantage of introducing the so-called *interphase element*, cf. Reedy and Mello (1996). Since automotive structures are largely modelled with shell elements, the simulation of adhesive joints between these shells will have to consider both the displacement and rotation of the shell adherends. Moreover, it has to span the distance from the centre of one of the shells to the centre of the next shell, i.e. the thickness of the adhesive layer plus the half thicknesses of the two shells, cf. Fig. 1(b) and paper B. Both these requirements are possible to account for in the interphase finite element formulation. The interphase element is a
finite element modelling of the adhesive layer and including the two adherends, thus claiming the appropriate name. In paper B, an interphase element is developed. Some commercial FE-codes provide methods for non-coinciding nodes on the two shells that are to be joined, cf. Fig. 5. We are therefore only considering coinciding nodes in paper B. Although coinciding nodes are not required across the joint, the element size or element size ratio may influence the properties. Mesh size effects are to be accounted for and have to be considered during the assessment phase of the correlation tests.

![Figure 5. Two adherends with non-coinciding nodes joined by interphase elements. Each adherend is meshed independently and connected to the interphase elements by contact interfaces of zero thickness in the 2-direction. For clarity, the contact interfaces are here depicted as grey striped bands of finite thickness.](image)

Some aspects of the interphase element formulation are investigated. An alternative method is also developed. In this model, the adhesive layer is modelled with continuum elements. Three continuum elements across the thickness of the adhesive layer are used to model the stress distribution appropriately. Moreover, the nodes of the adherends are connected to the nodes of the adhesive layer with rigid connections. The two types of models are compared. The interphase formulation shows a faster convergence than the continuum based model. Thus, the interphase formulation is much more efficient than the continuum formulation. The interphase element formulation shows a qualitatively correct stress profile with very large elements. In our verifying simulations, the time step size is slightly influenced by the interphase elements. However, in the continuum based element formulation there is a very strong influence, resulting in a substantially larger execution time. Furthermore, there is, in the continuum based formulation, a strong tendency of longitudinal stress waves propagating and reflecting through the adhesive layer. This effect is absent in the interphase formulation, since it lacks stiffness in this direction.

**Strain rate dependence**

Adhesives consist mainly of polymers, which are known to be strongly strain rate dependent. During an impact, automotive simulations show that the steel shells are subjected to high strain rates; up to about 300s⁻¹. It is not obviously the same rates that
appear in the adhesive. This has been investigated in paper A for a butt-joint. The study shows a much higher strain rate in the adhesive than in the base material. Since the nonlinear behaviour of the adhesive joint have to be triggered correctly, the stress distribution and corresponding strain-rates in an adhesive joint are very important to model accurately. In the present work, no investigation of the effects of a strain rate dependent cohesive law has been performed.

**Practical test specimen**

A simple test, which allows for all mode mixes from pure mode I to pure mode II, as well as various strain rates, is the ideal for testing adhesives. No such test method has been identified from the open literature. The idea is to model quasi-static tests of DCB or ENF, and see what happens if we run them dynamically. In paper A, a simple butt-joint serves as a simulation object and in paper B, a DCB test is simulated using both the commercial FE-software ABAQUS/Explicit, version 6.5 (ABAQUS Inc.) and an explicit MATLAB® code. The pure peel load in the DCB simulation is applied symmetrically in a soft manner and kept constant throughout the length of the test, exciting the first symmetrical eigenmode of the specimen.

**Conclusions**

Based on paper A, the conclusions are that the relatively high stiffness of the adhesive layer marginally reduces the critical time step; the reduction is less than 7 % for typical automotive data. Furthermore, the cohesive zone model accurately reproduces the basic characteristics of the wave propagation and reflection occurring in an adhesive joint. Strain rates in the adhesive layer are much higher than in the base material and in a butt joint, the relation of the strain rate in the adhesive, $\dot{\varepsilon}_a$, to the strain rate in the bar, $\dot{\varepsilon}_b$, is determined from the closed form solutions to be

$$\frac{\dot{\varepsilon}_a}{\dot{\varepsilon}_b} = \frac{E}{h\tilde{K}},$$

where $\tilde{K} = K/A$ is the adhesive layer stiffness, $E$ is Young’s modulus of the structure and $h$ is the thickness of the adhesive layer. As long as the relation $\tilde{K} > 12E/L$ is fulfilled, the stress state is virtually unaffected by the stiffness of the adhesive layer. This relation may be used to artificially decrease the stiffness of the adhesive layer and thus, increase the time step size. In this way, the influence of the adhesive layer stiffness on the critical time step size may be minimised.

In paper B, the governing equations for impact simulation and the finite element formulation are derived. An interphase formulation is presented for the 2D case of two shells joined by a thin adhesive layer. The mass matrix of the interphase element is derived and lumped. It is argued that material damping is of little importance relative to other dissipative mechanisms. Finally, a 2D verification simulation is performed on a pure peel DCB specimen. The performance with the interphase elements is compared to an identical structure conventionally modelled with continuum elements. Convergence and execution times are compared for the two
methods. The simulation using the interphase element formulation converges faster and is performed using larger time steps and fewer degrees of freedom, thus resulting in a significantly reduced execution time, cf. Eqs. (64, 66) in paper B. The time saving is considerable in the 2D case and even larger in the 3D case. It is concluded that the interphase formulation is superior to the continuum model regarding convergence and execution time. These properties are important in the context of modelling adhesive joints during impact of shell structures.

**Future work**
Due to the lack of experimental data on the strain rate dependence of adhesives, the obvious next step is to develop an experimental method. The interphase element and the cohesive model derived in the present work will be used to evaluate prospective specimens and techniques. Designing and instrumentation of an appropriate dynamic experiment is essential to the verification of this work.

The present work is focused on pure peel loading. In the future, shear loading and mixed mode loading will be included.

We mentioned earlier the importance of implementing strain rate dependence. The strain rate may be difficult to measure directly, but similar relations, as the one for the butt joint in paper A, may be derived, cf. Eq. (5).

In order to make the interphase element formulation useable in crash simulation, it is also necessary to investigate the influence of the element size and element size ratio.

There are different possibilities to implement this kind of cohesive elements for use in the field of adhesive modelling. The presented work does not claim to be complete. Commercial FE-code providers, such as ABAQUS and LS-Dyna both provide possibilities for the user to write their own user-defined routines which may offer other possibilities to model impact of adhesive joints efficiently.

**Acknowledgements**
The authors would like to express gratitude towards the Swedish Consortium for Crashworthiness for funding this project. Special thanks are also directed to Kent Salomonsson and Svante Alfredsson for fruitful discussions and help during the work of these papers.
References
ABAQUS Manuals v6.5. (2004), ABAQUS, Inc.
Dynamic fracture of adhesive joints using explicit FE-code
and the adhesive layer theory

T. Carlberger¹ and U. Stigh²
¹SAAB Automobile AB, SWEDEN and ²University of Skövde, SWEDEN

Abstract
Dynamic fracture of an adhesive layer in a structure is analysed. The structure
represents some specific properties of an automotive structure and is simple enough to
allow for closed form solutions. These solutions are compared to results of explicit FE-
analyses. The FE-solutions agree with the closed form solutions. Three constitutive
models of the adhesive layer are used. It is shown that an amplification of the strain rate
is achieved in the adhesive layer. It is also shown that an artificially increased flexibility
of the adhesive gives only minor influences of the general behaviour. In some load
cases, the adhesive layer will experience repeated loading/unloading. It is shown that in
these cases an explicit FE-analysis with a “large” time step is more prone to give
immediate rupture. Thus, the method is conservative.

Keywords: Adhesive joining; Dynamic fracture; Cohesive zone
Nomenclature

- $A$: bar cross sectional area
- $\alpha$: characteristic constant along characteristic line
- $\beta$: characteristic constant along characteristic line
- $c$: elastic wave speed
- $\varepsilon$: strain
- $\dot{\varepsilon}$: strain rate
- $E$: Young’s modulus
- $F$: applied load
- $G$: shear modulus
- $G_c$: fracture energy/energy release rate
- $h$: adhesive thickness
- $K$: adhesive layer stiffness
- $l_{\text{min}}$: shortest distance between two nodes
- $L$: element length, bar length
- $\nu$: Poisson’s ratio
- $\omega$: damage parameter
- $\omega_{\text{max}}$: largest eigenfrequency of the structure
- $\rho$: material density
- $\sigma$: peel stress
- $\dot{\sigma}$: ultimate cohesive zone strength
- $\tau$: shear stress
- $\tau$: characteristic time-parameter
- $T$: time for the wave to travel the bar length $L$
- $t$: time
- $\Delta t$: time step
- $u$: displacement
- $v$: shear deformation
- $v$: velocity of a material point
- $w$: peel separation
- $x$: coordinate direction
1. Introduction

In the automotive industry increasing requirements on emissions, cost and crash performance is driving a technology change from mono-material spot welded steel to multi-material adhesively joined car bodies. Thus, the automotive industry is increasing its focus on the use of adhesive joints due to benefits in strength, stiffness and the capability of joining dissimilar materials. One reason adhesive joining has not reached general acceptance, is a lack of reliable and efficient simulation methods for adhesives in the field of crash simulation. In the present work, an attempt is made to demonstrate a useful technique for this purpose.

Most crash simulations are based on explicit FE-codes since these are capable of simulating fast events within a reasonable execution time. Explicit FE-codes do not require the solution of systems of equations as in implicit FE-codes. Instead, the equation of motion of each degree of freedom is solved individually, cf. e.g. [1]. This allows for very large models such as complete and detailed automotive structures, to be analysed for impact studies. Such a model typically consists of over one million degrees of freedom. The limit on the degrees of freedom is due to limits in computer memory capacity and on the need to keep the solution time reasonable. To this end, the Courant limit (CFL-limit) is essential; in order to achieve numerical stability, the time step, $\Delta t$, must be smaller than $2/\omega_{\text{max}}$, where $\omega_{\text{max}}$ is the largest eigenfrequency of the structure. A useful estimate of the critical time step is given by the smallest time it takes an elastic wave to travel the distance between two nodes in the structure. This estimated critical time step $\Delta t_c$ is given by

$$\Delta t_c = \frac{l_{\text{min}}}{c} \quad \text{where} \quad c = \frac{E}{\sqrt{\rho}},$$

where $c$ is the wave speed, $l_{\text{min}}$ is the shortest distance between two nodes, $c$ is the wave speed, $E$ is the Young’s modulus and $\rho$ is the density of the material. If the time step is chosen larger than $\Delta t_c$ the FE-solution will fail due to numerical instability. With a typical time step $\Delta t$ of 1 $\mu$s, steel allows for a minimum length $l_{\text{min}}$ of about 5 mm and a typical adhesive material; epoxy has $l_{\text{min}} \approx 1.2$ mm. This should be compared to a
typical adhesive layer thickness of 0.2 mm. Thus, to achieve a stable numerical solution with an epoxy adhesive layer with 0.2 mm thickness, the time step has to be decreased to 0.165 µs. This increases the simulation time by a factor of six. Today, a typical simulation time for a complete car crash with the time step 1 µs is about 12 hours. With the smaller time step the simulation time increases to about 72 hours. This is not acceptable. A trick, often used in the automotive industry, is “mass scaling”. The material density is increased by a factor $n$ for the “too small elements” until the Courant limit is fulfilled with the desired time step. With Eq. (1) the scaling factor is

$$n = \frac{E}{\rho \left( \frac{\Delta \tau_c}{l_{\text{min}}} \right)^2}$$

(2)

This procedure adds extra mass to the model, about 30 to 40 kg on a complete car body. Obviously, this is a small extra weight as compared to the total mass of a car. However, the added mass is localized to the joints which might create an abnormal stress distribution in the joint during the severe accelerations during a crash simulation. Especially if the adhesive bond line is oriented in the crash direction, the added mass can create too high stresses in the surrounding regions. This is a problem today, since spot welds are simulated using this trick, [2]. In fact, when simulating spot welds, Young’s modulus is often reduced by an order of magnitude to avoid excessive mass scaling, clearly indicating that the localized masses can be a problem. Thus, the numerical problems encountered when joining sheet metal is not only limited to adhesive joining but appears to be a general problem when using commercial explicit FE-codes. Since the joints are not allowed to fracture during a crash, rigid links between the nodes at each sheet would appear as a good approach. This would lead to a reduction of the number of degrees of freedom. However, the risk for the joints to fracture cannot be determined using this approach.

A promising method to analyse adhesive joints is the use of the adhesive layer theory, cf. [3]. Based on an asymptotic analysis it is concluded that two deformation modes dominate the behaviour of a thin layer between stiffer adherends, cf. [4]. These
deformation modes are here denoted peel deformation, $w$, and shear deformation, $v$, cf. Fig. 1. The conjugated stresses are the peel stress, $\sigma$, and the shear stress, $\tau$.

![Diagram of deformation modes and conjugated stresses](image)

Figure 1: Deformation modes of the adhesive layer with thickness $h$: peel, $w$, and shear, $v$. Conjugated stress components $\sigma$ and $\tau$.

Experiments performed in pure peel and in pure shear on the commercial epoxy adhesive DOW Betamate XW1044-3 show similar behaviour in peel and sheer, cf. [5,6] and Fig. 2. The shear curve essentially appears as an enlarged version of the peel curve. As shown in the graphs, the traction-separation relations start with a linearly increasing part corresponding to linear elasticity. After reaching a peak stress, the stress decreases to zero stress after substantial deformation. As showed in [3,5,6], the area under the traction-separation relation equals the fracture energy. Thus, the present adhesive is substantially stronger in shear than in peel loading.

![Graph of constitutive behaviour in peel and shear](image)

Figure 2: Constitutive behaviour in peel and shear for the engineering adhesive DOW Betamate XW1044-3 with a 0.2 mm layer thickness, results from [5,6].
These results are confined to a low strain rate. A systematic analysis of the effect of strain rate is given in [7], cf. Fig. 3. Although, the strain rate is very small as compared to the strain rates encountered in crash simulations, the results indicate that the fracture energy varies with the deformation rate. Thus, a constitutive model for commercial epoxy adhesives should be rate dependent. A nonlinear viscoelastic model is suggested in [8].

![Figure 3: Fracture energy and maximum stress vs. rate of deformation in peel at $w = 20 \mu m$ for DOW Betamate XW1044-3 with a 0.2 mm layer thickness, results from [7].](image)

Experiments conducted at high strain rates involving adhesive joints are scarce in the literature. In [9] impact shear tests of adhesive layers are reported. In their setup they measure the average stress in an adhesive layer and conclude that fracture at low stress levels are caused by tensile stress while fracture at high stress levels are caused by a combination of shear and compression stress. In [10] the thick adherend shear test is used to determine the dynamic shear modulus of structural adhesives. The authors conclude that this method can be used for predicting the dynamic shear modulus if $G/h > 1 \text{ GPa/mm}$, where $G$ is the shear modulus of the adhesive and $h$ is the adhesive thickness. Since adhesive thicknesses are very small, this ratio will usually be exceeded by most commercial structural adhesives. In [11] a review of the standard ISO 11343, cf. [12] is performed. A correlation between results from the impacted wedge peel test (IWP) and the fracture mechanically determined fracture energies, $G_c$, of some adhesives is performed. It is shown that a finite-element analysis of the IWP test
A7

geometry may predict the failure behaviour successfully. In [13] a two-parameter criterion for fracture is suggested, involving both a critical limiting value of the stress, $\sigma_{\text{max}}$, and the critical energy release rate, $G_c$. The authors suggest that several parameters are involved in determining the fracture process. Including the aforementioned parameters, the damage zone length and the critical displacement, are included in such a way that each parameter is partly limiting.

Considering the similarities between adhesive joints and composite delamination, the field of search may be enlarged by including the latter. Strain rates or crack speeds associated with composite testing and simulation are generally much higher than for adhesives, due to the brittle nature of composites. In [14] dynamic mode-I delamination of a carbon/epoxy composite is studied in a modified DCB specimen. It is noted that the critical dynamic fracture energy is nearly constant and very close to the static fracture toughness of the composite. Numerical simulations performed by the authors show that the strain energy release rate at the straight crack front is non-uniform and dips towards the free edges. In [15] an investigation of the rate dependence of mode-I fracture toughness of carbon fibre epoxy laminates is reported. The results show that the crack growth behaviour change from unstable to stable due to differences in static and dynamic fracture toughness. In [16] mode-II-dominated delamination of fibre composites is studied achieving crack propagation speeds exceeding 1000 m/s.

The constitutive behaviour described by the graphs in Fig. 2 is only representative for a monotonically increasing deformation of the adhesive layer. In a crash simulation, some parts of the adhesive joint may experience such deformation. However, substantial parts of the joint will experience repeated loading and unloading. Thus, a constitutive theory for the adhesive layer should be capable of treating unloading as well as loading. In the present paper, we develop a continuum damage model for the adhesive layer. The model is implemented in an explicit FE-code and a numerical example is studied. The problem analysed is one which allows for a closed form and exact solution.
2. Simplified model

Studying the influence of a cohesive zone model during an impact situation in a complete vehicle body is a very complicated task. During a crash event, the first contact between the vehicle and the obstructing object results in an elastic wave being introduced in the car structure. This wave will quickly reach every corner of the structure and reflect back. Since the waves propagate through the steel structure at roughly 5000 m/s, the entire car structure will have been reached by the elastic wave in less than 1 ms. This time can be compared to the time for a crash, about 100 ms, cf. e.g. [17]. After the first passage of an elastic wave, there will be reflected waves continuously passing through the structure and it will be virtually impossible to follow each individual wave. Thus, all too many mechanisms are active at the same time to make substantiate conclusions possible. In the present paper we will therefore study a simplified problem that allows for some closed form solutions. In this way, numerical errors and potential problems can be singled out.

In spite of its simplicity, the impacted one-dimensional bar illustrates many of the phenomena encountered in an impact situation. Consider the system in Fig. 4. An elastic bar with length \( L \) and cross sectional area \( A \) is loaded at its right end at \( t = 0 \) with a constant load \( F = \sigma_0 A \), where different values of \( \sigma_0 \) will be analysed. The elastic modulus and density are \( E \) and \( \rho \), respectively. At the left hand side, the bar is connected to a rigid wall with a cohesive element that loads the bar with a force \( F_{\text{coh}} = A \sigma(w) \), where \( \sigma(w) \) is the traction-separation law.

![Figure 4: Simplified model.](image)

In order to simplify the real traction-separation relation shown in Fig. 2 the three step-wise linear traction-separation relations of Fig. 5 are suggested. The model in Fig. 5a
gives a good representation of the behaviour of an adhesive joint during moderate loading. Both this model and the linearly softening model in Fig. 5b allow for closed form solutions. However, the model in Fig. 5b is not useful for deformation based FE-analyses since the model has an infinite initial stiffness, i.e. before the joint starts to soften. The third model, cf. Fig. 5c, is in reasonable agreement with the experimental results in Fig. 2 and has been used extensively in studies of adhesive layers.

One-dimensional elastic wave propagation is governed by the well known wave equation,

$$\frac{\partial^2 u}{\partial \tau^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2},$$

where $c$ is the elastic wave speed given by Eq. (1b). We will first use the classical method of characteristics to develop two special solutions of the present problem, i.e. the solutions to the problem with a linear elastic or a linear softening traction-separation relation, cf. Figs 5a,b respectively. The first solution corresponds to a moderately loaded body, while the latter corresponds to an extensively loaded body in the case where the elastic properties of the joint can be neglected.

In order to simplify the present problem, the solution of Eq. (3) is evaluated along the characteristics. With $v$ denoting the velocity of a material point, i.e. $v = \partial u / \partial t$, the characteristics are given in $x-t$-space by straight curves determined by $dx/dt = \pm c$. Along
these curves, the solution of Eq. (3) is governed by ordinary differential equations. The solutions are given by

\[(\alpha): \quad \sigma - v\sqrt{E\rho} = \alpha \quad \text{at} \quad \frac{dx}{dt} = +c \quad (4a)\]

\[(\beta): \quad \sigma + v\sqrt{E\rho} = \beta \quad \text{at} \quad \frac{dx}{dt} = -c \quad (4b)\]

where \(\alpha\) and \(\beta\) are constants along respectively characteristic. The curves are denoted \(\alpha\)- and \(\beta\)-characteristics. Thus, the partial differential equation (3) is reformulated into two algebraic equations (4a,b). The solutions are conveniently separated in different regions in the \(x-t\)-space, cf. Fig. 6.

![Figure 6: Left: Regions in the \(x-t\)-space; Right: Details of region C](image)

Along the boundary, \(t = 0\), the initial conditions \(\sigma = v = 0\) hold and along the boundary, \(x = L\), the boundary condition \(\sigma = \sigma_0\) holds. At the left boundary, \(x = 0\), the stress is given by the traction separation relation, \(\sigma = \sigma(w)\), where the relation between \(w\) and \(v\) is given by \(v(0,t) = \frac{dw}{dt}\) and \(w = u(0,t)\). When the load is applied at the right hand side at \(t = 0\), an elastic wave with velocity \(c\) propagates from the right hand side to the left. At time \(T = L/c\) the wave hits the adhesive layer. Thus, in the lower triangular region in the \(x-t\)-space, i.e. the region A, both the stress and the velocity are zero. In region B, the behaviour is governed by the stress free region below and the stressed boundary to the right. Along a \(\alpha\)-characteristic in region B, Eq. (4a) holds. The three un-known, \(\sigma\), \(v\) and
\( \alpha \), are determined by the conditions at the boundaries to these two regions. At the boundary to the region A, both \( \sigma \) and \( v \) are zero and Eq. (4a) gives the constant \( \alpha = 0 \). At the right hand side, the stress equals \( \sigma_0 \). Thus, Eq. (4a) gives the velocity

\[
v = \frac{\sigma_0}{\sqrt{E\rho}},
\]

(5)

in region B; in region B, both the stress, \( \sigma = \sigma_0 \) and the velocity are constant. Now, depending on the specific traction-separation relation, specific solutions are derived after this point, i.e. in region C.

2.1 Linear elastic adhesive layer

In region C, the behaviour of the adhesive layer influences the results. With a low level of the applied load, the adhesive layer behaves as a linear elastic medium. Thus, the boundary condition at \( x = 0 \) is

\[
\sigma(0,t) = Kw(t),
\]

(6)

where \( K \) corresponds to the modulus of elasticity of the adhesive layer. A \( \beta \)-characteristic, Eq. (4b), in region C is governed by the conditions at the boundaries to the adhesive layer at \( x = 0 \) and at the boundary to region B. From the conditions at the boundary to region B, the constant is determined to \( \beta = 2\sigma_0 \). The conditions at the boundary to the adhesive layer yield the following ordinary differential equation for the separation,

\[
\frac{dw}{dt} + \frac{1}{\tau_e} \frac{w}{K} = \frac{2\sigma_0}{\tau}
\]

(7)

where \( \tau_e = \sqrt{E\rho / K} \) is a characteristic time-parameter for the problem. Solving Eq. (7) with the initial condition, \( w = 0 \) for \( t = T \) yields,
\[ w(t) = \frac{2\sigma_0}{K} \left[ 1 - \exp\left( \frac{T-t}{\tau_e} \right) \right] \]  \hspace{1cm} (8)

The corresponding velocity and stress are given by differentiation with respect to \( t \) and multiplication of \( w(t) \) with \( K \), respectively. The result is

\[ v(0,t) = \frac{2\sigma_0}{\sqrt{E\rho}} \exp\left( \frac{T-t}{\tau_e} \right) \quad \text{and} \quad \sigma(0,t) = 2\sigma_0 \left[ 1 - \exp\left( \frac{T-t}{\tau_e} \right) \right] \]  \hspace{1cm} (9a,b)

To obtain the stress and velocity at \((x,t)\) we now use the property of Eqs. (4a,b) to hold along a \( \alpha \)- and \( \beta \)-characteristic, respectively. Thus,

\[ \begin{align*} 
(\alpha): & \quad \sigma(0,t-\Delta t) - v(0,t-\Delta t)\sqrt{E\rho} = \sigma(x,t) - v(x,t)\sqrt{E\rho} \\
(\beta): & \quad \sigma(0,t+\Delta t) + v(0,t+\Delta t)\sqrt{E\rho} = \sigma(x,t) + v(x,t)\sqrt{E\rho}
\end{align*} \]  \hspace{1cm} (10a,b)

where \( \Delta t \) is the time it takes a wave to travel to/from the adhesive layer to the point \( x \), cf. Fig. 6. Thus, \( \Delta t = x/c \). With Eqs. (9a,b), Eqs. (10a,b) constitute a linear system of equations for \( \sigma \) and \( v \), the solution is

\[ \begin{align*} 
\sigma(x,t) &= 2\sigma_0 \left[ 1 + \exp\left( \frac{T-t+x/c}{\tau_e} \right) \right] \quad \hspace{1cm} (11a) \\
v(x,t) &= \frac{2\sigma_0}{Kr_e} \exp\left( \frac{T-t+x/c}{\tau_e} \right) \quad \hspace{1cm} (11b)
\end{align*} \]

This way of stepping forward one region at a time can be used as long as desired, though the expressions grow more and more complex as the solution proceeds.

2.2 Linear softening adhesive layer

With a large load and with a stiff adhesive layer, the softening behaviour of the layer is expected to dominate the behaviour. In this case, a linear softening model is suitable, cf.
Fig. 5b. The solution method is similar to the one described for the elastic layer. As compared to the solution above, a softening adhesive layer results in a change of Eq. (6) to
\[\sigma(0,t) = \bar{\sigma} \left( 1 - \frac{w}{w_c} \right),\] (12)
where the symbols are defined in Fig. 5b. Proceeding as described above, the solution in region C is given by
\[
\sigma(x,t) = 2\sigma_0 - \bar{\sigma} \left( 2\frac{\sigma_0}{\bar{\sigma}} - 1 \right) \exp \left( \frac{t-T-x/c}{\tau_c} \right)
\] (13a)
\[
v(x,t) = \frac{w_c}{\tau_c} \left( 2\frac{\sigma_0}{\bar{\sigma}} - 1 \right) \exp \left( \frac{t-T-x/c}{\tau_c} \right)
\] (13b)
where the characteristic time-parameter is now changed to \(\tau_c = w_c \sqrt{E\rho / \bar{\sigma}}\). These closed-form solutions give insight into the different length- and time-scales of a solution. The closed-form solutions will also be compared to the results of FE-simulations.

A more realistic traction-separation law should have the characteristics indicated in Fig. 5c. Moreover, during a crash simulation, multiple waves will travel back and forth through a structure. Thus, the models of the adhesive joints must be able to handle unloading from a loaded state.

2.3 Unloading and damage
Indications from in situ tensile tests in a SEM suggest that an adhesive layer will develop only small amounts of plastic strain during peel loading, cf. [18]. Thus, most of the in-elastic deformation is attributed to the development of microscopic cavities. These are expected to close upon unloading. A simple constitutive model of this process is provided by the introduction of a damage variable, \(\omega\), cf. [19]. The damage parameter \(\omega\) is defined through \(\sigma = (1-\omega)Kw\), cf. Fig. 7. Suppose the specimen is loaded to point
A, with $w_1 < w < w_c$. At this point, unloading starts. Since we have exceeded $w_1$, the stiffness at unloading is $(1-\omega)K$. At the next loading, the stiffness is still $(1-\omega)K$ until the point A is reached. After this point has been passed, softening proceeds until either next unloading occurs or $w_c$ is passed and the cohesive zone fails completely.

![Figure 7: Cohesive zone model with damage $\omega$ and unloading.](image)

**2.4 Explicit FE-code**

For comparisons an explicit FE-code is developed, cf. e.g. [1,20]. The code is developed in MATLAB and provides an estimated critical time step, cf. Eq. (1a), and also the maximum eigenfrequency.

**3. Analyses and results**

In this section we will present some numerical results based on the closed form solution and the FE-analysis. At moderate loading, the response of the adhesive layer is linear elastic and Eqs. (8,9) provide the exact solution in region C where the behaviour of the adhesive layer is first influencing the results. A typical result is given in Fig. 8. At $x = 0$, the wave first hits the layer at $t = T$. The stress and displacement then gradually build up. At $x = L/2$, the elastic wave first passes on its way to the adhesive layer at $t = T/2$; at $t = 3T/2$ the reflected wave returns. The sudden stress-dip at $t = 3T/2$ is due to the limited stiffness of the adhesive layer.
Figure 8: Stress (left) and displacement (right) response with a linear elastic adhesive layer. Solid line at $x = 0$ and dashed line at $x = L/2$. Data for $T = \tau_e$.

With an infinitely stiff layer, the stress in region C will be twice the applied load, i.e. $\sigma = 2\sigma_0$. Indeed, with $K \to \infty$, $\tau_c \equiv \sqrt{E\rho / K} \to 0$, and Eq. (11b) yields $\sigma = 2\sigma_0$ for $T < t \leq 3T$. As indicated above, the effect of the flexibility of the adhesive layer is to immediately reduce the stress at the left end to zero at loading. However, with a large stiffness of the adhesive layer, the stress rapidly increases to $2\sigma_0$. Now if a lower stiffness can be chosen without influencing the results in any substantial way, the critical time step can be increased, cf. Eq. (1). As a reasonable criteria, the stress can be claimed to be only marginally influenced by the flexibility of the adhesive layer if $\sigma > 1.8\sigma_0$, at $t = 1.2T$, i.e. only a $10\%$ lower stress than with an infinitely stiff layer attained within $10\%$ of the loading time. Evaluation of Eq. (9b) yields the condition

$$ K > 12 \frac{\sqrt{E\rho}}{T} = 12 \frac{E}{L}, $$

As long as this condition is fulfilled, the stress state is virtually un-affected by the stiffness of the adhesive layer. For a typical structural length, $L = 0.2$ m, the stiffness can be chosen as low as $12$ TPa/m (steel) and $4.2$ TPa/m (Al). These values can be compared to $K = 20$ TPa/m for a typical epoxy adhesive layer cf. e.g. [3]. Thus, the time step can be substantially increased with a decrease of the elastic stiffness of the adhesive
layer in an aluminium structure. However, the effect is only marginal for a steel structure.

The maximum stress in the adhesive layer is attained at $t = 3T$. Equation (11a) gives

$$\sigma_{\text{max}} = 2\sigma_0 \left(1 - \exp\left(-\frac{2T}{\tau_c}\right)\right) < 2\sigma_0.$$  

(15)

If $\sigma_{\text{max}} \leq \tilde{\sigma}$, no inelastic deformation takes place in the adhesive layer. However, if the loading is large and if the stiffness of the adhesive layer is large, the linear softening model of Fig. 5b should provide a good estimate. The solution Eq (13a) reveals that the adhesive layer will fracture during the first loading period, $T < t < 3T$, if

$$\sigma_0 \geq \sigma_{0c} = \frac{1}{2} \tilde{\sigma} \frac{\exp(2T/\tau_c)}{\exp(2T/\tau_c) - 1} \approx \frac{1}{2} \tilde{\sigma} \quad \text{if} \quad T \geq T_c,$$  

(16)

where $T_c \approx \tau_c/2$, or equivalently, if $L > Ew_c / 2\tilde{\sigma} \approx 0.3$ m (steel) to 0.1 m (Al) where data for the adhesive layer is chosen according to Fig. 2. If this condition is fulfilled, the applied load has to exceed only half the adhesive strength for the layer to fail during the first loading period, i.e. about 10 MPa for the adhesive in Fig. 2. The stress and displacement response for a load case where the adhesive layer fails during the first loading period is given in Fig. 9. The first wave hits the adhesive layer at $t = T$ and the stress increases immediately to $\sigma = \tilde{\sigma}$. At the same time, the adhesive layer starts to elongate and the stress falls off. At $x = L/2$, the first wave passes at $t = T/2$. At $t = 3T/2$, the wave returns after being reflected by the adhesive layer.
We will now make comparisons with the results of FE-simulations. The simplified problem in Fig. 2 is modelled with a linear elastic adhesive layer and simulated with an explicit FE-code. Data is chosen according to $L = 0.2$ m, $E = 210$ GPa, $\rho = 7800$ kg/m$^3$ and $K = 20$ TPa/m, respectively. The bar is divided into 100 elements of equal length. Thus, the time it takes a wave to pass one element is $0.39 \mu s$ and the time to pass the bar is $T = 39 \mu s$. The characteristic time-parameters are $\tau_c = 2.0 \mu s$ and $\tau_e = 61 \mu s$.

Figure 10: Stress and displacement comparison between the FE-method (FE) and the closed form solution (C.M.). The FE-simulation is evaluated with the time step $\Delta t = 0.367 \mu s$, which is 95% of the critical time step for a system without a cohesive zone.
Figure 10 shows the stress and displacement history. As expected, the displacements of the two methods agree excellently. In Fig. 10 another artefact becomes evident. At the end of the bar, subjected to the step load, considerable numerical noise is induced in the simulation due to the steep gradient of the step load. Thus, the FE-method is unable to reproduce abrupt transients. The conventional method to temper this artefact is to gradually increase the applied load to its final value. Figure 11 shows the stress and displacement history with an elastic linear-softening traction-separation law.

Figure 11: Stress and displacement history with an elastic linear-softening traction-separation law including damage. The FE-simulation is evaluated with the time step $\Delta t = 0.367\,\mu s$, which is 95 % of the critical time step for a system without an adhesive layer.

As evident from Fig. 11, the stress response resembles both the closed form solutions. At the time the wave hits the adhesive layer, at 39 $\mu s$, the stress gradually builds up to the maximum stress, 20 MPa, at about 120 $\mu s$. During this time interval, the solution resembles the solution with a linear elastic adhesive layer, cf. Fig. 8. After this point, the stress decreases similarly as in Fig. 9. The adhesive layer breaks at about 160 $\mu s$ and the bar accelerates as a free bar to the right.

In spite of these inherent drawbacks of the FE method, we consider these results successful. Excellent agreement is achieved between the closed form solution, Eqs (11)
and the numerical results for the elastic layer. The conclusion is that the finite element model is capable of simulating the global characteristic behaviour of the impacted bar with a satisfactory engineering accuracy.

Upon impacting of a car structure, stress waves are induced in the impacted area and transmitted throughout the complete vehicle structure. Since the wave speed of steel exceeds 5000 m/s the whole vehicle will be traversed by stress waves in less than one millisecond. A crash event has a duration of around 100 ms. During this time, the induced and reflected stress waves will damage and fracture different structural parts. It is plausible to assume that many adhesive joints will be repeatedly loaded and unloaded during this event, causing an accumulation of damage in each adhesive region subjected plastic deformation. This scenario may be simulated by adjusting the step load level such that rupture occurs after several waves. If the cohesive force, $F_{coh} = \sigma(w)A$, is plotted against the separation during this event, a plot according to Fig. 12 results.

As evident in Fig. 12, the unloading follows the reduced stiffness $(1-\omega)K$. Depending on the ratio between the applied load and the cohesive strength, $\sigma_0 / \bar{\sigma}$, the behaviour differs. With a large load, fracture occurs during the first loading period. A smaller load leads to repeated loadings and unloadings until the layer fractures. Numerical
experiments show that a certain range of load levels exist during which repeated loadings and unloadings occur without final fracture. Thus, the accumulation of damage stops even though the loading continues. Numerical experiments also show that this behaviour is critically dependent on the chosen time step, cf. Fig. 13.

As shown in Fig. 13, a large time step leads to fracture after some loadings and unloadings. With a short time step, giving a more accurate solution, the damage accumulation rests. Thus, a simulation of this effect will be conservative; a longer time step will more likely predict fracture than a short time step.

In spite of these results, one should be aware of the difficulties for the explicit FE-method to deal with constitutive relations with abrupt changes. The cohesive force is calculated from the deformation at the previous time step, which results in a force delay depending on the time step size used. A smaller time step gives a more accurate evaluation of the cohesive force than a large time step. Due to this, the calculated dissipated energy from this cohesive zone behaviour will not be accurately predicted for multiple unloadings and a larger time step.

Polymers, which constitute the bulk material in an adhesive, are known to be very strain rate dependent. Crash analysts in the automotive industry observe strain rates in the
base material around 300 s\(^{-1}\) in simulations, [2]. Although very noisy, strain rates tend to be high in the sheet metal. This clearly justifies the requirement of strain rate dependant constitutive models for the sheet metal. The strain rate dependency increases the yield strength more than 100 % in low grade steel. High strength steel does not show this strong dependency. Therefore, an interesting question is how high the strain rate in the adhesive will be. The strain rate in the adhesive layer calculated in the FE-simulations is presented in Fig. 14. If the load level is low, the joint will not fracture. But if the load exceeds a certain level, the joint will fracture after several reflections of the stress wave. Further increase of the load leads to joint fracture after consecutively fewer stress wave reflections, until fracture occurs after just one reflection. Note that the time scales are equal in both graphs.

![Figure 14: Strain rate for joint fractured after multiple (left) or a single (right) stress wave reflection.](image)

The strain rate is slightly above 2000 s\(^{-1}\) for the multiple wave rupture and around 4000 s\(^{-1}\) for the single wave rupture. Test data is not generally available for adhesives or polymers up to these levels of strain rate, but it is reasonable that this is of great importance in order to be able to simulate rupture of adhesive joints accurately. The strain rate in the adhesive can be compared to the strain rate in the bar which is of the order 1-100 s\(^{-1}\). Thus, the present geometry leads to a substantial amplification of the strain rate from the structure to the adhesive layer. Note that the cohesive zone model
used in this analysis is not strain rate dependent. A strain rate dependent cohesive zone would influence the achieved strain rate.

4. Conclusions
The adhesive layer model with different traction-separation relations has been analysed with the method of characteristics and also implemented in an FE-code to simulate an adhesive joint. The analyses show that the relatively large stiffness of the adhesive layer gives a shortened critical time step. However, the exact value of the stiffness has a minor effect on the solution if the stiffness is chosen larger than about $12E/L$, cf. Eq. (14). This indicates a good possibility to adjust an FE-model to achieve a reasonable execution time for aluminium structures. However, for steel structures additional measures have to be taken in order to achieve a reasonable time step. If the applied load is larger than about half the strength of the adhesive layer, the layer will start to soften, cf. Eq. (15). Furthermore, if the load is sufficiently large, the adhesive layer will fracture during the first loading period, cf. Eq. (16). It is also showed that the general behaviour can be reproduced with an explicit FE-code. The examples show that the characteristics of the wave propagation and reflection behaviour are adequately handled by the cohesive zone model. During repeated loading, the development of damage may stop. It is argued that this property is dependent on the time step used in the simulations. Numerical examples do however show that a short time step is more likely to predict a dormant damage. Thus, an explicit FE-simulation will be conservative in this respect. Moreover, the simulations indicate a very large strain rate in the adhesive layer. From the closed form solutions, Eqs (8,11,12,13), the ratio of the strain rate in the adhesive layer to the strain rate in the bar can be derived. The ratios estimated to $E/hK \approx 50$ and $Ew_c/\dot{\sigma}h \approx 1600$ for the elastic and the softening layer, respectively; the numerical values corresponds to the adhesive layer considered in this paper. Thus, the strain rate is expected to be several orders of magnitude larger in the adhesive layer than in the bar for the present joint geometry. It is however concluded that a more realistic traction-separation law should include strain-rate dependence.
Acknowledgements
The authors would like to acknowledge the Swedish Consortium for Crashworthiness for funding this project. Special thanks are also directed to Kent Salomonsson and Svante Alfredsson for fruitful discussions and help during the work of this paper.

References


[18] Salomonsson K, Andersson T. Modeling and parameter calibration of an adhesive layer at the meso level. 2006; Submitted for publication.


Explicit FE-formulation of Interphase Elements for Adhesive Joints

Thomas Carlberger\textsuperscript{1} and Ulf Stigh\textsuperscript{2}

\textsuperscript{1}SAAB Automobile AB, SWEDEN and \textsuperscript{2}University of Skövde, SWEDEN

Abstract
The potential of adhesive bonding to improve the crashworthiness of cars is attracting the automotive industry. Large-scale crash simulations are time consuming when using the very small finite elements needed to model an adhesive joint using conventional technique. In the present work, a 2D-interphase element formulation is developed and implemented in an explicit FE-code. A simplified joint serves as a test example to compare the interphase element with a straightforward continuum approach. A comparison is made of the execution time and shows the time saving potential of the present formulation as compared to the conventional approach. Moreover, the interphase element formulation shows fast convergence and computer efficiency.

Keywords: Interphase element; Dynamic fracture; Adhesive joint
Nomenclature

A  metric matrix
\(a_{1-12}\) nodal degrees of freedom of interphase element
\(a_i(t)\) displacement and rotation of node \(I\) at time \(t\)
\(B\) specimen width
\(b\) volume load
\(c_I\) vector with arbitrary elements associated with node \(I\).
\(\Delta t, \Delta t_c\) time step and critical time step
\(\Omega, \partial \Omega\) body and outer surface
\(\delta_{ij}\) Kronecker delta
\(\delta\) separation vector of adhesive layer
\(E\) Young’s modulus
\(f_I\) force vector associated with node \(I\)
\(F\) applied load vector
\(G\) matrix in the relation between \(\delta\) and \(a\)
\(g\) essential boundary conditions
\(h\) thickness of adhesive layer
\(H_1, H_2\) thickness of lower and upper adherend, respectively
\(I, J\) node numbers
\(J^{\text{cons}}, J^{\text{lump}}\) moment of inertia from consistent (cons) and lumped (lump) mass matrix
\(\kappa\) wave number
\(\xi, \zeta\) local coordinates of adhesive layer
\(L, L_b, L_s\) element length, bonded length, and specimen length
\(l_0\) distance between two zero-values of the peel stress
\(M\) mass matrix
\(m\) mass of adhesive in one element
\(n\) outward normal
\(n_{\text{st}}\) number of time steps
\(N_I\) shape function associated with node \(I\).
\(\omega_{\text{max}}\) largest eigenfrequency of the structure
\(\dot{\omega}\) angular acceleration
\(R\) virtual power associated with the adhesive layer
\(r\) adherend number
\(\rho\) material density
\(S\) interface surface
\(\sigma\) Cauchy stress
\(T_r, T_s\) computer execution time and total time to simulate
\(t\) time
\(t_{\text{ev}}\) evaluation time for one degree of freedom and one time step
\(\bar{t}\) prescribed traction vector
\(\Theta\) adherend cross-sectional rotation
\(u\) displacement vector
\(v, w\) shear and peel deformation of adhesive layer
\(v\) displacement and rotation of adherend mid-line
\(v\) weight function
\(v_{a,i}\) Poisson’s ratio of adherend (index a) and adhesive (index i)
\(x\) coordinate
1. Introduction

Car bodies consist of large shell structures connected with some bonding technique. Traditionally, the most frequent technique is spot-welding. Although this method has many advantages, it is essentially limited to mono-material joints. Enhanced demands on fuel efficiency and lowered emissions increase the need for further optimisations. To this end, a multi-material car body is a promising possibility. Recently the potential to use adhesive bonding has been identified. With this method it is not only possible to join dissimilar materials but also to improve both stiffness and strength in mono-material structures.

In the product development phase, explicit FE-simulations are frequently used to evaluate the crash worthiness of prospective car bodies, cf. e.g. [1]. In these simulations, there is a need for efficient modelling technique of adhesive joints. Specifically it is necessary to model the structure in such a way that the simulation execution time, $T_r$, is kept as short as possible; typically not exceeding 12-15 hours. An inherent limit with the explicit FE-method is the Courant limit; in order for an explicit FE-simulation to be numerically stable, the time step $\Delta t$ must be smaller than $2/\omega_{\text{max}}$, where $\omega_{\text{max}}$ is the largest eigenfrequency of the structure, cf. e.g. [2]. This critical time step can be estimated as the shortest time it takes an elastic wave to travel the distance between two nodes. In practice, this critical time step sets a limit on the smallest element length for a given time step and total execution time. This critical element length is for an adhesive consisting mainly of epoxy about 1.2 mm with a time step $\Delta t = 1$ µs. This element length should be compared to the typical adhesive layer thickness, $h = 0.2$ mm. Thus, modelling the adhesive layer with continuum finite elements will increase the execution time by a factor of about six, cf. [1]. A trick, often used to circumvent the Courant limit, is mass scaling or a combination of mass scaling and a reduction of Young’s modulus, [3]; the material density is increased and/or Young’s modulus of the adhesive is decreased for the “too small elements” until the Courant limit is fulfilled. This method is to be used with caution since the added mass or decreased stiffness will influence the results. Especially if the adhesive bond line is oriented along the crash direction, added mass may give too large influences on the results. In a recent study of the butt-joint, a 40 % reduction of the
stiffness of the adhesive layer is shown to be tolerable, [1]. These types of results should however be taken with some caution since they are confined to specific joint geometries and load cases.

In FE-simulations, the large shell structures of the car body are modelled with shell elements. A convenient method to model adhesive bonds of shell structures is to use interphase elements, cf. [4]. These elements connect the degrees of freedom of the shell element with the deformation of the adhesive layer, i.e. the peel deformation \( w \) and the shear deformation \( v \), cf. [5]. In the present paper, an interphase element is developed for the adhesive joint and compared to the results of a conventionally modelled continuum based adhesive joint model. It is shown that an FE-simulation with interphase elements provides a more efficient analysis than the continuum based simulation.

2. Interphase formulation

Figure 1 shows a part of an adhesive joint consisting of two shells. In this case the shells are the two parts creating the B-pillar of a passenger vehicle. Typical dimensions are adhesive layer thickness \( h = 0.2 \) mm and thickness of shells \( H_1 \approx H_2 = 0.8 \) mm. Thus, we often find \( h < H_1 \approx H_2 \). The bonded sheets are usually modelled with shell elements in the explicit FE-formulation. This means that the displacement field in the shells is governed by nodal displacements and rotations in the FE-model. In order to connect the thin adhesive layer correctly to the shells, the coupling of the rotational degrees of freedom of the shells to the displacement of the adhesive layer has to be considered, cf. [5]. A simple method to achieve this coupling is to model the adhesive as a continuum using solid elements and to connect these to the shell elements by the use of rigid bars or stiff beam-elements, cf. Fig. 1.
Figure 1: 3D-model of adhesively joined sheets (adherends). The adhesive layer is represented by a solid element connected to the shell elements by rigid bars.

In the case of two-dimensional models, the shells are modelled as beams and the adhesive layer may be modelled with a four-node isoparametric solid element as indicated in Fig. 2b. The adhesive element is connected to the adherends by rigid bars or stiff beams. Although this is a tedious work, the process may be automated. In this case, the use of rigid beams is preferred, since it does not add degrees of freedom to the system. The stiff beams will add degrees of freedom to the system, but more severely, they will negatively influence the critical time step, $\Delta t_c$, of the system. Since the adhesive is modelled as a continuum, the very thin thickness of the adhesive layer will, referring to the Courant limit, imply a correspondingly short time step. An attractive alternative method to avoid this tedious work and not severely influencing the critical time step, is to use the interphase element discussed in [6], cf. Fig. 2b. Obvious profits of modelling the adhesive with interphase elements are easier modelling and shorter execution time as will be exemplified in this paper. Next, we will derive the governing equations in the FE-problem and the consistent mass matrix.
2.1 Governing equations

Consider a body $\Omega$ with outer surface $\partial \Omega$. Conservation of linear momentum in the body $\Omega$ is given by

$$\sigma_{ij,j} + \rho b_i - \rho \ddot{u}_i = 0 \quad \text{in} \quad \Omega. \quad (1)$$

where $\sigma$ is the Cauchy stress tensor, $\rho$ is the density, $b$ the volume load, $u$ the displacement vector and dots are used to indicate differentiation with respect to time, $t$. Here, lowercase indexes indicate Cartesian coordinates, $i, j \in \{1, 2, 3\}$; all components are taken with respect to a common inertial frame. Moreover, summation is to be taken over repeated indexes. On the parts of the boundary $\partial \Omega$ where the traction is prescribed the following equation applies

$$\sigma_{ij} n_j = \bar{T}_i \quad \text{on} \quad \partial \Omega. \quad (2)$$

where $\bar{T}$ is the prescribed traction vector. On other parts of $\partial \Omega$, the displacement and velocity might be prescribed. To obtain the weak form of Eq. (1), we multiply with an arbitrary weight function $\nu$ and use the divergence theorem, cf. e.g. [7]. Omitting some intermediate steps, the result is
\[ \int_{\Omega} \nu_i \sigma_{ij} \, d\Omega - \int_{\Omega} \nu_i \rho \beta_i \, d\Omega - \int_{\Omega} \nu_i \bar{t}_i \, d(\partial\Omega) + \int_{\Omega} \nu_i \rho \bar{u}_i \, d\Omega = 0 , \]  

which is known as the principle of virtual power. In order to FE-formulate this equation we introduce

\[ \mathbf{u} = N_I a_I , \]  

where the indexes in capital letters indicate the node numbers, i.e. \( I = 1,2,\ldots,n_{\text{nodes}} \), and summation should be taken over repeated indexes in capital letters. Here \( a_I(t) \) is the displacement/rotation of node \( I \) and \( N_I \) is the shape function associated with node \( I \). The weight function \( \nu_i \) is approximated according to the Galerkin-method, i.e. using the same shape functions as are used for the displacements

\[ \mathbf{v} = N_I \mathbf{e}_I \]  

where \( \mathbf{e}_I \) is a vector with arbitrary elements associated with node \( I \). Now, substituting Eq. (5) into the principle-of-virtual power, Eq. (3) and noting that the \( \mathbf{e}_I \)'s are arbitrary we arrive at

\[ \int_{\Omega} \frac{\partial N_I}{\partial x_j} \sigma_{ij} \, d\Omega - \int_{\Omega} N_I \rho \beta_i \, d\Omega - \int_{\partial\Omega} N_I \bar{t}_i \, d(\partial\Omega) + \int_{\Omega} N_I \rho \bar{u}_i \, d\Omega = 0 \]  

With Eq. (4) we arrive at

\[ f^\text{int}_I - f^\text{ext}_I + M \ddot{a}_I = 0 , \]  

where

\[ f^\text{int}_I = \int_{\Omega} \frac{\partial N_I}{\partial x_j} \sigma_{ij} \, d\Omega , \quad f^\text{ext}_I = \int_{\Omega} N_I \rho \beta_i \, d\Omega - \int_{\partial\Omega} N_I \bar{t}_i \, d(\partial\Omega) , \]  

\[ f^\text{kin}_I = \int_{\Omega} N_I \rho \bar{u}_i \, d\Omega = \int_{\Omega} \rho N_I N_J \bar{d} \bar{\overline{u}}_{ij} , \]  

and

\[ M_{ij} = \delta_{ij} \int_{\Omega} \rho N_I N_J \, d\Omega . \]
Here, $\delta_{ij}$ is the Kronecker delta. Equation (7) is a set of second order ordinary differential equations.

$$\mathbf{M} \ddot{\mathbf{a}}_I = \mathbf{f}_I^{\text{ext}}(\mathbf{a}, \dot{\mathbf{a}}, t) - \mathbf{f}_I^{\text{int}}(\mathbf{a}, \dot{\mathbf{a}}, t) \equiv \mathbf{f}_I .$$

(9)

With a diagonal mass matrix, Eq. (9) constitutes the set of equations of the explicit FE-method. The procedure to diagonalise the generally non-diagonal mass matrix is known as lumping, cf. e.g. [2]. Essential boundary conditions complement Eq. (9). These are imposed on $n_c$ degrees of freedom, i.e.

$$\mathbf{g}_I(\mathbf{a}, \dot{\mathbf{a}}, t) = 0, \ I = 1, 2, ..., n_c,$$

(10)

where the nodal degrees of freedom $\mathbf{a}_I$ have been collected in the vector $\mathbf{a}$. The elements of $\mathbf{a}$ are usually nodal displacements and, for beam and shell formulations, nodal displacements and rotations. Similarly, the elements of $\mathbf{f}$ are associated with nodal forces and for beam and shell elements, with nodal forces and moments. These equations constitute the foundation for the development of an interphase element in the next subsection.

### 2.2 The interphase element

In this section we will derive the nodal forces and the mass matrix associated with the interphase element. We will also discuss the properties of damping associated with an adhesive layer. Consider an arbitrary point on the adherend $r$, $r \in \{1, 2\}$ according to Fig. 3. Here $r = 1$ denotes the lower adherend and $r = 2$ the upper one. Let $\mathbf{a}^{(r)}$ denote the degrees of freedom of the adherend $r$. According to Fig. 2, the vectors are given by

$$\mathbf{a}^{(1)} = [a_1, a_2, a_3, a_4, a_5]^T, \quad \text{and} \quad \mathbf{a}^{(2)} = [a_7, a_8, a_9, a_{10}, a_{11}, a_{12}]^T$$

(11)

Furthermore, let $\mathbf{v}^{(r)}$ denote the displacement and rotation of a point along the mid line of adherend $r$. For a beam element, this vector contains the two mid line displacements $u_x^{(r)}$ and $u_y^{(r)}$ in $x$- and $y$-direction, respectively and the cross-sectional rotation, i.e. $\Theta_z^{(r)}$.
\[ \mathbf{v}^{(r)} = \begin{bmatrix} u_x^{(r)} & u_y^{(r)} & \Theta_z^{(r)} \end{bmatrix}^T. \] (12)

The local degrees of freedom \( \mathbf{v}^{(r)} \) are interpolated from the element nodal degrees of freedom, \( \mathbf{a}^{(r)} \), by

\[ \mathbf{v}^{(r)} = \mathbf{N}^{(r)} \mathbf{a}^{(r)} \] (13)

Figure 3: Beam degrees of freedom, \( \mathbf{v}^{(r)} \) and displacements \( \mathbf{u}^{(r)} \) on the interface surfaces.

For a Mindlin beam element, the interpolations of the displacements and rotation of the middle line are independent, i.e.

\[ \mathbf{v}^{(1)} = \begin{bmatrix} u_x^{(1)} \\ u_y^{(1)} \\ \Theta_z^{(1)} \end{bmatrix} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_6 \end{bmatrix} = \mathbf{N}^{(1)} \mathbf{a}^{(1)} \] (14)
where the nodal shape functions are given by \( N_1 = 1 - \xi \) and \( N_2 = \xi \) with \( \xi \) given in Fig. 3. Now, let \( \mathbf{u}^{(r)} = \begin{bmatrix} u_x^{(r)} & u_y^{(r)} \end{bmatrix}^T \) be the displacement on the interface between the adherend \( (r) \) and the adhesive. Figure 3 shows

\[
\mathbf{u}^{(r)} = \mathbf{A}^{(r)} \mathbf{v}^{(r)}
\]

where

\[
\begin{align*}
\mathbf{A}^{(1)} &= \begin{bmatrix} 1 & 0 & -H_x/2 \\ 0 & 1 & 0 \end{bmatrix} \\
\mathbf{A}^{(2)} &= \begin{bmatrix} 1 & 0 & H_x/2 \\ 0 & 1 & 0 \end{bmatrix},
\end{align*}
\]

are metric matrices. The displacement of the interface boundary, \( \mathbf{u}^{(r)} \), is obtained from Eqs. (13) and (15):

\[
\mathbf{u}^{(r)} = \mathbf{A}^{(r)} \mathbf{N}^{(r)} \mathbf{a}^{(r)}.
\]

The separation of the interface surfaces \( S^{(1)} \) and \( S^{(2)} \), i.e. the deformation of the adhesive layer, \( \mathbf{\delta} \), is obtained from Eq. (17) as

\[
\mathbf{\delta} = \mathbf{u}^{(2)} - \mathbf{u}^{(1)} = \begin{bmatrix} -\mathbf{A}^{(1)} \mathbf{N}^{(1)} & \mathbf{A}^{(2)} \mathbf{N}^{(2)} \end{bmatrix} \mathbf{a} \equiv \mathbf{G} \mathbf{a}
\]

whereby the matrix \( \mathbf{G} \) is defined. The total separation of the adherends is given by the components \( \delta_x \) = deformation of the adhesive layer in the \( x \)-direction and \( \delta_y \) = deformation in the \( y \)-direction. This forms the separation vector \( \mathbf{\delta} = \begin{bmatrix} \delta_x & \delta_y \end{bmatrix}^T \). By use of the orientation of the adhesive layer, the separation vector can be decomposed in the peel and shear components, respectively.

Comparing with Eq. (17), weight functions, \( \mathbf{v}^{(r)} \), are chosen according to the Galerkin method as

\[
\mathbf{v}^{(r)} = \mathbf{A}^{(r)} \mathbf{N}^{(r)} \mathbf{c}^{(r)}
\]

where \( \mathbf{c}^{(r)} \) contains arbitrarily chosen nodal values. Now, form the virtual power associated with the action of the adhesive on the adherends, \( R \), by

\[
B10
\]
\[ R = - \int_{S^{(\ell)}} \left( \mathbf{v}^{(2)} - \mathbf{v}^{(1)} \right)^T \mathbf{t} \, dS \]  

(20)

where \( \mathbf{t} \) is the traction acting on the adhesive on the adhesive/adherend interface \( S^{(\ell)} \), cf. Eq. (3). With, Eq. (19)

\[ \mathbf{v}^{(2)} - \mathbf{v}^{(1)} = \mathbf{Gc} \text{ where } \mathbf{c} = \begin{bmatrix} \mathbf{c}^{(1)} & \mathbf{c}^{(2)} \end{bmatrix}^T, \]  

(21)

and we arrive at

\[ R = -\mathbf{c}^T \int_{S^{(\ell)}} \mathbf{G}^T \mathbf{t} \, dS \equiv -\mathbf{c}^T \mathbf{F} \]  

(22)

where the integral is identified as the contribution to the nodal force vector from the adhesive layer. We may express the integrand in this expression by the use of Eq. (18) as

\[ \mathbf{G}^T \mathbf{t} = \begin{bmatrix} -\mathbf{N}^{(1)^T} \mathbf{A}^{(1)^T} \\ \mathbf{N}^{(2)^T} \mathbf{A}^{(2)^T} \end{bmatrix} \mathbf{t} \]  

(23)

leading to

\[ \mathbf{F} = \begin{bmatrix} \mathbf{F}^{(1)} & \mathbf{F}^{(2)} \end{bmatrix}^T, \text{ where } \mathbf{F}^{(I)} = \int_{S^{(I)}} \left( \mathbf{A}^{(I)} \mathbf{N}^{(I)} \right)^T \mathbf{t} \, dS \]  

(24a,b)

These forces are used in the explicit FE-formulation.

We will now derive the mass matrix, Eq. (8d), associated with the interphase element. Let \( \zeta \) denote a through the thickness coordinate in the adhesive layer with \( \zeta = 0 \) at the lower interface and \( \zeta = 1 \) at the upper interface, cf. Fig. 4.
The displacement $\mathbf{u}$ in the adhesive at a point $\zeta$ is written

$$\mathbf{u} = \begin{bmatrix} \mathbf{A}^{(1)} \mathbf{N}^{(1)} (1 - \zeta) \\ \mathbf{A}^{(2)} \mathbf{N}^{(2)} \zeta \end{bmatrix} \mathbf{a} = \mathbf{N}_{\text{el}} \mathbf{a},$$  \hspace{1cm} (25)$$

and the element mass matrix, Eq. (8d), is derived in a straightforward manner. The result is

$$\mathbf{M} = \int_{V} \rho \mathbf{N}_{\text{el}}^T \mathbf{N}_{\text{el}} dV = \frac{\rho B h L}{6} \int_{0}^{1} \begin{bmatrix} 2 \mathbf{N}^{(1)T} \mathbf{A}^{(1)T} \mathbf{A}^{(1)N}^{(1)} & \mathbf{N}^{(1)T} \mathbf{A}^{(1)T} \mathbf{A}^{(2)N}^{(2)} \\ 2 \mathbf{N}^{(2)T} \mathbf{A}^{(2)T} \mathbf{A}^{(1)N}^{(1)} & 2 \mathbf{N}^{(2)T} \mathbf{A}^{(2)T} \mathbf{A}^{(2)N}^{(2)} \end{bmatrix} d\xi$$  \hspace{1cm} (26)$$

Evaluation gives the consistent mass matrix

$$\begin{bmatrix} 16 & 0 & -8H_1 & 8 & 0 & -4H_1 & 4 & 0 & 2H_2 & 8 & 0 & 4H_2 \\ 16 & 0 & 0 & 8 & 0 & 0 & 4 & 0 & 0 & 8 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4H_1^2 & -4H_1 & 0 & 2H_1 & -2H_1 & 0 & -H_1H_2 & -4H_1 & 0 & -2H_1H_2 \\ 16 & 0 & -8H_1 & 8 & 0 & 4H_2 & 4 & 0 & 2H_2 \\ 16 & 0 & 0 & 8 & 0 & 0 & 4 & 0 & 0 \end{bmatrix}$$  \hspace{1cm} (27)$$

Since we will use the mass matrix in an explicit FE-code, this matrix has to be transformed to a diagonal matrix to allow the simple numerical scheme in the explicit FE-code. The literature provides several methods of lumping, cf. e.g. [2,7]. In a
crashworthiness analysis, essentially rigid body accelerations are anticipated in large parts of the structure. Thus, we lump the mass matrix so that a rigid body motion gives the correct momentum. Accordingly, the adhesive mass, \( m = \rho B h L \), is evenly distributed to the four nodes. By this procedure \( m/4 \) is placed at each of the four nodes and on the diagonal of the lumped mass matrix at the positions associated with the translational degrees of freedom. For the rotational degrees of freedom, we initially regard the adhesive as four rigid bars of weight \( m/4 \) and length \( L/2 \) rotating around each corner node, resulting in a rotational moment of inertia at each node respectively. The lumped mass matrix for the interphase element becomes

\[
M_{\text{Lumped}} = \frac{\rho B h L}{48}
\]

For a comparison of the momentum, we impose rigid-body accelerations. The lumped mass matrix gives the correct translational momentum. However, an imposed angular acceleration does not give the correct angular momentum. Suppose an interphase element is accelerated around its centre by the angular acceleration \( \dot{\omega} \). The interphase element inertial moment, \( f_3^{\text{kin}} \), calculated using the consistent mass matrix, Eq. (27), is

\[
\frac{\text{cons}}{} f_3^{\text{kin}} = \frac{\rho B h L}{12} \left( h^2 + L^2 \right) \dot{\omega} = J_{\text{adhesive}} \dot{\omega}
\]
which is identical to the result for a rigid body of length $L$, height $h$, width $B$, and density $\rho$ accelerated around the centre, cf. Fig 5.

![Figure 5: Adhesive, as rigid body, rotating around centre.](image)

The suggested lumped mass matrix, Eq. (28), gives the rigid body inertial moment

$$\text{lumped} J_{3}^{\text{kin}} = \dot{\omega} \frac{\rho BhL}{24} \left(3H_{1}^{2} + 6H_{1}h + 6h^{2} + 8L^{2} + 3H_{2}^{2} + 6H_{2}h \right) \equiv J_{\text{adhesive}}^{\text{lumped}} \dot{\omega}. \quad (30)$$

As evident, there is a discrepancy as $J_{\text{cons}}^{\text{adhesive}} \neq J_{\text{lumped}}^{\text{adhesive}}$. This is because the rotational moment of inertia is too large in the lumped mass matrix. The reason is that the positioning of the mass of the adhesive at the nodes gives a larger moment of inertia even if rotational inertia is neglected completely. This is because the mass of the interphase element is lumped and placed at the nodes. This fault may be corrected for by a negative rotational inertia in the lumped mass matrix.

The need for an improvement of the mass matrix is dependent on the ratio between the mass matrix of the adherend and the mass matrix of the adhesive. The mass matrix for a Mindlin beam element is

$$M_{\text{Mindlin}} = \frac{\rho_{r}BH_{r}L}{72} \begin{bmatrix} 24 & 0 & 0 & 12 & 0 & 0 \\ 24 & 0 & 0 & 12 & 0 \\ 2H_{r}^{2} & 0 & 0 & H_{r}^{2} & 0 \\ 24 & 0 & 0 & 0 & 0 \\ S & Y & M. & 24 & 0 \\ 2H_{r}^{2} & \end{bmatrix}, \quad r \in \{1, 2\} \quad (31)$$

This beam mass matrix can be compared to the mass matrix of the interphase element. To this end, form the “collected” mass matrix of the two beam elements connecting to the interphase element, i.e.
\[
\mathbf{M}_{\text{Mindlin}} = \begin{bmatrix}
\mathbf{M}_{\text{Mindlin}}^1 & 0 \\
0 & \mathbf{M}_{\text{Mindlin}}^2 
\end{bmatrix}
\] (32)

To study the importance of modelling the interphase mass matrix correctly, we form the ratio between the inertia of the adherend and the interphase. With typical values the relations in Table 1 result.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adhesive thickness (h)</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>Adherend thickness (H_1 = H_2)</td>
<td>0.8 mm</td>
</tr>
<tr>
<td>Adhesive density (\rho)</td>
<td>1350 kg/m(^3)</td>
</tr>
<tr>
<td>Adherend density (\rho_1 = \rho_2)</td>
<td>7800 kg/m(^3)</td>
</tr>
<tr>
<td>Element length (L)</td>
<td>5 mm</td>
</tr>
<tr>
<td>(J_{\text{lumped adhes.}}/J_{\text{cons adhes.}})</td>
<td>4.1</td>
</tr>
<tr>
<td>(J_{\text{lumped adher.}}/J_{\text{lumped adhes.}})</td>
<td>46</td>
</tr>
</tbody>
</table>

Table 1: Ratio between lumped rotational inertia, \(J_{\text{lumped adhes.}}\), and consistent rotational inertia, \(J_{\text{cons adhes.}}\), for the interphase element and ratio between adherend lumped inertia, \(J_{\text{lumped adher.}}\), and interphase element lumped inertia, \(J_{\text{lumped adhes.}}\), with typical data for an automotive application.

As Table 1 clearly shows, the mass matrix of the interphase element is much smaller than the mass matrix of the shell element. For the numerical stability of the explicit method, increased mass is positive, and thus we will not ignore the adhesive mass completely. Therefore we will use the lumped mass matrix of Eq. (28) for the interphase element.

Damping is also positive for the numerical stability of the explicit FE-method. Damping in structures is due to mechanisms such as hysteresis in the material and slip in connections. These mechanisms are not well understood. A popular assumption is damping due to viscous mechanisms. However, the amount of dissipated energy due to viscous damping is generally negligible as compared to the energy “dissipated” by the plastically deforming adherends in a crash analysis. Energy is also dissipated in the fracture process. However, this energy is also negligible in comparison to the energy lost due to plastic deformation of the adherends. With the present formulation, this energy will appear in the formulation since it is given by the area under the traction-separation relation for the adhesive layer, i.e.
\[ J_c = \int_0^\infty t_i d\delta_i \quad (33) \]

where \( t \) is the traction vector and \( \delta \) the separation of the adherends. Thus, the stress in the adhesive is a function of both the separation \( \delta \) and the rate-of-separation \( \dot{\delta} \). An example is given in Fig. 6. Accordingly, we write

\[ t = t(\delta, \dot{\delta}). \quad (34) \]

After rupture, we may assume that the adhesive will be fractured by cohesive mechanisms and leave about half the adhesive thickness on each of the adherends. This will lead to a damping effect too.

![Traction-separation behaviour for the engineering epoxy adhesive XW1044-3](image)

Figure 6: Traction-separation behaviour for the engineering epoxy adhesive XW1044-3 with layer thickness \( h = 0.2 \) mm. Solid curve: low strain rate; dashed curve: higher strain rate. Data from [8].

### 3. Verifying simulations

The interphase formulation is tested in a 2D explicit FE-code of a DCB structure subjected to symmetric dynamic loading, cf. Fig. 7. The purpose of the study is to evaluate how well the simplified interphase formulation agrees with a more involved continuum approach. Moreover, the convergence properties will be evaluated, i.e. the number of elements needed to achieve an acceptable level of error is evaluated. The
beams are modelled with linear elastic, linearly interpolated Mindlin beam elements and the adhesive with linear elastic, linearly interpolated interphase elements, cf. Eqs. (24,28). For comparison, an identical DCB-model is developed using conventional linear elastic, linearly interpolated Mindlin beam elements for the adherends and linear elastic, four-node rectangular continuum elements for the adhesive. The adhesive continuum elements are connected to the adherends by means of rigid connections. Thus, the rotational degrees of freedom of the beam elements are connected to the shear deformation of the adhesive layer as described above. The adhesive layer is modelled with three elements over the thickness to provide a possibility for some stress distribution through the thickness of the adhesive layer. This model is evaluated with the commercial FE-software ABAQUS/Explicit, version 6.5-1, ABAQUS Inc. Providence, USA. All models are in plane deformation.

Figure 7: (a) DCB modelled with interphase elements, and (b) DCB modelled with continuum adhesive and rigid connections. The layer thickness is \( h = 0.2 \) mm.

The DCB structure is given typical automotive dimensions: specimen length, i.e. \( L_s = 30 \) mm, adherend height \( H_1 = H_2 = H = 0.8 \) mm, adhesive layer thickness \( h = 0.2 \) mm.
and the bonded length $L_b = 15$ mm. The material of the adherends is steel with Young’s modulus $E_a = 210$ GPa, density $\rho_a = 7800$ kg/m$^3$, and Poisson’s ratio $\nu_a = 0.3$. The adhesive material is epoxy with Young’s modulus $E_i = 2.0$ GPa, density $\rho_i = 7800$ kg/m$^3$, and Poisson’s ratio $\nu_i = 0.4$. A symmetric force $F = 333$ N/m is applied in a smooth manner. This is achieved by gradually increasing the force during a time interval corresponding to the primary eigenfrequency in bending. After this time, the force is held constant. For comparisons, the peel stress along the adhesive layer is evaluated at the moment the first maximum deflection of the loading points is passed. In the continuum simulation, the peel stress in the centre of the adhesive layer is evaluated, i.e. in the middle layer of elements, cf. Fig. 7. The reason for this choice is the singularity at the corner of the adhesive layer and the adherend. Thus, if we would evaluate the stress at the interface, no convergence can be expected. Convergence is studied as the mesh size is decreased. A relevant length parameter in this context is the wave number, $\kappa \equiv 4\sqrt{\frac{6E}{E_a H^3 h}}$, that emerges from a quasistatic analysis, cf. e.g. [10]. Here,

$$\bar{E} = \frac{E_i (1-\nu_i)}{(1+\nu_i)(1-2\nu_i)}; \quad (35)$$

is the elastic stiffness of the adhesive layer considered to be constrained from deforming in the plane of the layer. The wave number has a simple interpretation; with $l_0$ denoting the distance between two neighbouring zero-values of the peel stress, $\kappa l_0 = \pi$. Thus, it is expected that some finite elements have to be used to adapt to the stress distribution along $l_0 = \pi/\kappa \approx 3$ mm.

The stress profiles with both the interphase and the continuum elements are plotted in Fig. 8. Figure 8a shows that the converged value of the peak peel stress is achieved already with $\kappa L = 2.4$, where $L$ is the element length. Thus, with the element length 2.3 mm a correct maximum peel stress is achieved. However, although the stress distribution resembles the converged one, it appears too coarse. Using half that element length, $\kappa L = 1.2$ gives a result that appears more adequate. Thus, using more than about three elements to resolve the stress distribution between two zero-values of the peel stress appears adequate for many applications. When comparing with the results using continuum elements in Fig 8b, it is obvious that the converged maximum
peel stress is different. The reason is the singularity at the corners between the adherends and the adhesive and the looser constraint at the free edge of the adhesive layer. In the interphase model, the stiffness of the layer is taken as the one corresponding to a fully constrained stiffness, cf. Eq. (35). The too stiff adhesive layer at the start of the adhesive layer in the interphase model only influences the behaviour of the adherends marginally. Thus, the deformation of the adherends is virtually the same in the continuum and interphase models. The larger stiffness in the interphase model at the free edge of the adhesive thus leads to a larger maximum stress than in the continuum model. Thus, the interphase model is conservative.

Figure 8. Stress profiles acquired with consecutively smaller elements with (a) interphase formulation and (b) continuum formulation.
An alternative method to handle the discrepancy between the continuum and interphase models is to require that the material behaviour is evaluated according to a specific theory. Thus, if a continuum model is to be used, the experiments used to measure the material behaviour should be evaluated considering the full field of stress and deformation in the adhesive layer. On the other hand, if the interphase model is to be used, only the peel and shear stress and deformation of the layer should be considered. This later method appears adequate and preferable, cf. e.g. [11].

The continuum model requires an element length $\kappa L = 0.15$, to resolve the stress distribution. That is one fourth of the element length required by the interphase model. The maximum peel stress, at the adhesive front of the interphase elements, converges for the element length $\kappa L = 2.4$, which is compared to $\kappa L = 0.31$ for the continuum elements, i.e. a factor of eight finer, which for the 3D case means that the number of elements required for simulation of an adhesive joint with continuum elements is 64 times larger. The maximum peel stress at the adhesive front is plotted against the normalised element length $\kappa L$ for both the interphase and continuum models in Fig. 9.

![Figure 9: Convergence study. The interphase element model converges faster than the model with continuum elements.](image)

The continuum based simulations suffer from longitudinal stress waves overlaying the peel stress and also, to some extent, from hour-glassing. Neither of these deficits
plague the interphase element, since the element is fully integrated and lacks stiffness in the longitudinal direction.

Apart from providing an easier geometrical modelling as compared to continuum elements, the interphase element gives a shorter execution time. With $T_s$ denoting the total time to be simulated, the number of time steps $n_M = T_s/\Delta t = T_s c/L$, where the time step is related to the wave speed $c$ and the element length according to the estimated Courant limit, cf. e.g. [1]. Thus, the smaller elements which are necessary using continuum elements, lead to an increased number of time steps. With a specific object to be analysed, the length of the elements and the number of nodes are related to the size of the object. For the models of the DCB-specimen analysed here, the number of pairs of beam elements along the bond line equals $L_b/L$, cf. Figs. 7b and 10. With the interphase element, each pair of beam elements contributes with six degrees of freedom (DOF) to the total number of DOF; six extra DOF add at the last element pair. With the continuum elements, using three elements through the thickness of the adhesive layer, each pair of beam elements contributes with ten DOF and ten extra at the last element pair, cf. Fig. 10. With $t_{ev}$ denoting the evaluation time for one degree of freedom and for one time step, the total execution time for an analysis $T_r = 6t_{ev} T_s c/L(L_b/L+1)$, where the factor 6 is changed to 10 for the continuum model. Moreover, the element length $L$ for a converged analysis for the continuum model is one eighth of that for the interphase model. Thus, the relation between the execution time with the interphase model to the execution time for the continuum model is about 0.009. A similar analysis for a 3D case gives 0.001 see Fig. 11. These numbers apply to structures constituting the adhesive joint. In automotive structures where the adhesive joints constitute a minor part of the complete structure, the numbers are larger.
Figure 10. Principal sketches of 2D adhesive joints modelled by (a) interphase elements and by (b) continuum elements.

Figure 11. Principal sketches of 3D adhesive joints modelled by (a) interphase elements and by (b) continuum elements.

Consider a structure where one percent of the total degrees of freedom (DOF) connect to adhesive joints, which are modelled with interphase elements. If we substitute the adhesive elements for continuum elements, the total number of DOF will have to increase due to the slower convergence of this element type. The relation between the
total numbers of DOF using a continuum formulation as compared to the numbers of DOF using the interphase formulation is 1.95. Since the time step size is only one eighth in the continuum case the execution time increases with a factor 14.6. This is not acceptable for engineering use.

4. Results and conclusions

The governing equations for impact simulation and the finite element formulation are derived. An interphase formulation is presented for the 2D case of two shells joined by a thin adhesive layer. The interphase mass matrix is derived and lumped. It is shown that the material damping matrix is of little importance relative other dissipative mechanisms. Finally, a 2D verification simulation is performed on a pure peel DCB specimen using the interphase elements representing the adhesive joining the two beam adherends and compared to an identical structure conventionally modelled with continuum elements representing the adhesive and connected to the two beams by rigid connections. The comparison not only involves the convergence of the two techniques but also their respective total execution times. The simulation using the interphase element formulation converges faster and can be evaluated using larger time steps and fewer degrees of freedom, thus rendering a significantly reduced execution time. It seems that the interphase formulation may be an attractive technique of modelling adhesive joints during impact of shell structures. It should be noted that there is a possibility to partition the structure into two parts; with one part containing the joints some special technique can be used for this part. Either smaller time steps are used, i.e. subcycling [7,9,12], or an implicit scheme is used for this part together with the explicit method for the rest of the structure, cf. e.g. [13].

Acknowledgements

The authors are grateful to Dr Svante Alfredsson, Mr Anders Biel and Mr Kent Salomonsson for fruitful discussions during this study. The authors would also like to acknowledge the Swedish Consortium for Crashworthiness for funding this project.
References:


