ABSTRACT

An experimental method to determine the complete stress versus deformation relation for a thin adhesive layer loaded in shear is presented. The work is based on a classical specimen geometry, \textit{i.e.} the end-notch flexure specimen (ENF-specimen) and the experiments are evaluated based on an inverse method. By studying the energy balance at the crack tip an expression for the energy release rate is derived. The theory considers the effects of a flexible adhesive layer and is based on beam theory. From the energy release rate the stress-deformation relation is derived using the inverse method.

Quasi-static experiments are performed using a servo-hydraulic testing machine. The deformation process at the crack tip is monitored during the experiments by use of a CCD-camera attached to a microscope. The method requires differentiation of the energy-deformation relation, therefore a Monte-Carlo simulation is performed to investigate how small errors in the data acquisition system affects the final stress-deformation relation. Small errors in the measurement of the force and shear deformation give small effects on the final stress-deformation relation.

Experiments on three different geometries of the specimen are performed. The experiments give consistent results. It is shown that if the process zone in front of the crack tip is large, than the stress-deformation relation does not depend on the dimensions of the adherends. Thus, the constitutive relation can be considered to be a property of the adhesive layer.

**Keywords:** adhesive layer, shear, ENF-specimen, stress-deformation relation, experimental method, \textit{J}-integral, inverse method
PREFACE

This work has been carried out at the Division of Mechanical Engineering at the University of Skövde, Sweden during the years 2002 - 2005. The work has been done in cooperation with the Department of Applied Mechanics at the Chalmers University of Technology, Gothenburg, Sweden.

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**NOMENCLATURE**

- **a** crack length
- **b** distance between the crack tip and loading point
- **k** elastic stiffness of the adhesive
- **p** length of the process zone
- **t** thickness of the adhesive layer
- **u** longitudinal displacement
- **v** shear deformation in the adhesive layer
- **v_0** shear deformation at the crack tip
- **v_c** shear deformation at fracture
- **w** deflection
- **E** Young’s modulus of the adherend
- **E_ad** Young’s modulus of the adhesive
- **G** Shear modulus of the adhesive
- **H** height of the specimen
- **J** energy release rate
- **J_ad** simplified form of energy release rate
- **J_c** fracture energy
- **L** length of the specimen
- **M** bending moments
- **N** normal force
- **P** applied force
- **P_c** force at fracture
- **U** complementary energy
- **V** transversal force
- **W** width of the specimen
- **ε** strain
- **κ** wave number
- **ν_ad** Poisson’s number of the adhesive
- **σ** normal stress
- **τ** shear stress in the adhesive layer
- **τ_J** Jouravski’s shear stress
- **Δ** displacement of the loading point
- **Π** potential energy
1 INTRODUCTION

Due to the increasing demands for lightweight structures, the possibility to use the optimal material for each part of a structure needs to be utilised. This leads to a growing interest in adhesive joining since this method gives greater possibilities to join dissimilar materials as compared to more traditional methods such as riveting, bolting and welding. In many application areas, it is advantageous to use adhesives together with for example spot welding. This provides structural integrity during the assembly process before the adhesive is cured.

Adhesive joining has additional advantages, e.g. it provides some vibration isolation, it gives galvanic isolation and it gives smaller shape distortions than welding. To exploit all the advantages of adhesive joints, they have to be designed properly. If the same design as used for riveting and welding is used, the optimal properties of adhesive joints are not utilised. It is well known that adhesive joints can carry much larger loads in shear than in peel, cf. Kinloch (1987). It is therefore important to design the adhesive joint so that it is primarily exposed to shear stress. However, an adhesive joint is always hyperstatic and the stress distribution depends on the constitutive properties of the adhesive. The present work is focused on determining the shear characteristics of an adhesive layer and it complements work by Andersson and Stigh (2004) and Andersson and Biel (2004) where the properties in peel are determined.

It is common to use linear elastic fracture mechanics (LEFM) together with an energy based approach to predict the strength of adhesive bonds with adherends deforming elastically, cf. e.g. Papini et al. (1994), Fernlund et al. (1994) and Högberg (2004). How well LEFM works depends on the possibility to disregard the fracture process taking place in the adhesive layer. If the process zone in front of the crack can be considered as small as compared to the other length parameters in the problem, the energy release rate, $J$, can be used to predict the strength of the joint. But since many engineering adhesives are ductile and will deform substantially before fracture, the process zone will be long, cf. e.g. Andersson and Stigh (2004), Leffler and Stigh (2005). Therefore, in many cases, LEFM can not be expected to work well to predict the strength of adhesive joints.
A better approach can be to use a constitutive law to predict the strength of adhesive joints. If such laws exist, they can be used to predict both stress distribution and strength of adhesive joints irrespective of the behaviour of the adherends. Thus, the same law should be applicable for elastically and plastically deforming adherends. The law should also be applicable to different adherends, e.g. a law determined by use of steel adherends should be useful with aluminium adherends. These types of models are used by, e.g. Stigh (1988), Yang and Thouless (2001), and Kafkalidis and Thouless (2002), to predict the strength of adhesive joints.

The two dominating deformation modes in adhesive layers are peel and shear according to an asymptotic analysis by Klarbring (1991), cf. Fig. 1. A method to determine the two simple constitutive relations, $\sigma(w)$ and $\tau(v)$, are suggested by Olsson and Stigh (1989) in peel and by Alfredsson (2003) in shear. These laws are derived using an inverse method and are valid for monotonically increasing deformation. A similar method is used to determine constitutive laws for delamination of composites in peel (Fernberg and Berglund, 2001), for adhesive layers in peel (Andersson and Stigh, 2004; Sørensen, 2002) and adhesive layers in shear (Alfredsson et al. 2003; Leffler and Stigh, 2005). The constitutive laws for peel and shear are shown in Fig. 2 for the present adhesive (DOW Betamate XW1044-3). Another way to determine these laws is suggested by Yang et al. (1999) for peel and Yang et al. (2001) for shear. They measure the force-displacement curves experimentally and adjust numerical simulations to the experimental data by assuming the Tvergaard-Hutchinson cohesive law, cf. Tvergaard and Hutchinson (1992).

The present constitutive relations are no constitutive laws in an ordinary sense because they also depend on structural parameters. Since the fracture energy of an adhesive layer depends on the thickness of the layer (Chai, 1988) the $\tau$-$v$-relation is also a function of this structural parameter. Analyses by Nairn (2000) show that the apparent fracture toughness of an adhesive
layer depends on the residual stresses. This indicates that the constitutive relation also depends on these.

In the present paper the stress-deformation relation is determined for a thin adhesive layer loaded in shear. The stress-deformation relation is assumed to be a unique relation valid along the adhesive layer. To derive the stress-deformation relation a classical specimen geometry is utilised, *i.e.* the end-notch flexure specimen (ENF), *cf.* Fig. 3. This specimen is used by *e.g.* Carlsson *et al.* (1986) and Chai (1988) to study both brittle and ductile adhesive layers.

**Figure 2** Constitutive behaviour in peel and shear for the engineering adhesive DOW Betamate XW1044-3 with a 0.2 mm layer thickness. Peel: Andersson and Stigh (2004).

In the present paper the stress-deformation relation is determined for a thin adhesive layer loaded in shear. The stress-deformation relation is assumed to be a unique relation valid along the adhesive layer. To derive the stress-deformation relation a classical specimen geometry is utilised, *i.e.* the end-notch flexure specimen (ENF), *cf.* Fig. 3. This specimen is used by *e.g.* Carlsson *et al.* (1986) and Chai (1988) to study both brittle and ductile adhesive layers.

**Figure 3** The ENF-specimen.

The ENF-specimen has some important advantages compared to for example butt joints and single lap shear specimens, *cf.* Fig. 4. Stable crack growth which can be achieved with the ENF-specimen is difficult to achieve for the butt joint. Therefore, it is difficult to capture the complete stress-deformation relation using a butt joint. In the ENF-specimen the stress distribution in the adhesive layer is essentially pure shear except at the loading point where some compressive stresses appear. In the single lap joint a uniform shear stress can not be guaranteed, *cf.* *e.g.* Tsai and Morton (1994) and Högberg (2004).
In the ENF-specimen the state of stress in the adhesive layer is inhomogeneous. The stress-deformation curve can therefore not be directly measured. Thus, an inverse method is used. The method utilised in the present paper requires that the adherends deform elastically. The energy release rate is measured experimentally and the shear stress is derived by using an inverse method. According to Rice (1968), the $J$-integral is path independent and $J$ is equal to zero if the domain inside the closed path does not contain any singularity. The integral is given by

$$J = \int_S \left( W_s dy - T_i u_i dS \right),$$

where $W_s$ is the strain energy density, $T_i$ is the traction vector defined according to the outward normal to the counter clockwise curve $S$, and $u_i$ is the displacement vector. The $J$-integral is path independent if the material is hyperelastic. If the material is inelastic, $J$ is still path independent if no unloading takes place from an inelastic state. Thus, the $J$-integral can be used in this case.

When evaluating the $J$-integral at the crack tip, $J$ takes the value

$$J(v) = \int_0^v \tau(\tilde{v}) d\tilde{v}.$$  

Thus, if $J$ can be measured as a function of $v$ during an experiment, differentiation of Eq. (2) gives the $\tau$-$v$-relation.

$$\tau(v) = \frac{\partial J(v)}{\partial v}$$

It has been argued that the relation may depend on the thickness of the adherends. The reason might be that thicker adherends give larger process zones and thus an altered fracture process. This idea will be studied in the present paper.
2 THEORY

The objective of this work is to find the complete stress-deformation relation for a thin adhesive layer loaded in shear. This is achieved by using the inverse method described in the introduction where the stress is given by the derivative of the energy release rate with respect to the shear deformation. Thus, the energy release rate needs to be determined. In this section, the energy release rate is derived for three models of the adhesive layer: a stiff adhesive layer, a linear elastic adhesive layer and a general adhesive layer. The specimen used for this purpose is the end-notch flexure-specimen (ENF), cf. Fig. 5. As described in the introduction, almost no normal stresses appear in the adhesive layer in the ENF-specimen. It is also possible to design a specimen which provides stable crack propagation. This is discussed later in Section 3.

For a homogeneous, elastic material the energy release rate is given by,

\[
J = -\frac{1}{W} \frac{\partial \Pi}{\partial a}
\]

where \( \Pi \) is the potential energy of the body and its loading and \( W \) is the width of the specimen. The ENF-specimen with a ductile adhesive layer can be analysed as a beam on an inelastic foundation for the adherend. This gives the following differential equations,

\[
\tau(x) = \begin{cases} 
\frac{EH}{8} v''(x) + \bar{\tau} & 0 \leq x \leq b \\
\frac{EH}{8} v''(x) - \bar{\tau} & b \leq x \leq L - a 
\end{cases}
\]

Figure 5  The geometry of the ENF-specimen. The out of plane width of the specimen, \( W \)
Equation (5) is valid for elastic Euler-Bernoulli beams and for small deformations and is used to derive an expression for the $J$-integral for both a linear elastic and general inelastic adhesive layer in Sections 2.2 and 2.3, respectively.

### 2.1 Rigid adhesive layer

The energy release rate for an ENF-specimen with a rigid adhesive layer is derived. The adherends are only allowed to deform elastically according to the Euler-Bernoulli beam theory. The potential energy, $\Pi$, as a result of bending of the beams for a prescribed load, $P$, is given by

$$\Pi = \frac{1}{64} \frac{P^2}{EWH^3} \left(12a^3 + L^3\right)$$

This gives, using Eq. (4),

$$J_0 = \frac{9}{16} \frac{P^2a^2}{EW^2H^3}$$

which is derived by e.g. Russell and Street (1982).

### 2.2 Linear elastic adhesive layer

For any test specimen, the energy release rate is the area under the stress-deformation curve, cf. Eq. (2). With a linear elastic adhesive, the shear stress can be expressed as $\tau = kv$, cf. Fig. 6, and the energy release rate is given by

$$J_{el} = \frac{k\nu^2}{2a^2}$$
where \( v_0 = v(0) \) is the shear deformation at the crack tip, \( x = 0 \), cf. Fig. 5, and \( k \) is the elastic stiffness of the adhesive, \( k = G/t \), where \( G \) is the shear modulus of the adhesive and \( t \) is the thickness of the adhesive layer. Thus, the deformation at the crack tip needs to be determined.

The shear deformation along the adherend is given by the solution to the differential equation in Eq. (5) with a linear elastic adhesive layer. The solution is

\[
v(x) = \begin{cases} 
A_1 e^{\kappa x} + A_2 e^{-\kappa x} + \frac{\tau}{k} & \text{if } 0 \leq x \leq b \\
A_3 e^{\kappa x} + A_4 e^{-\kappa x} - \frac{\tau}{k} & \text{if } b \leq x \leq L - a 
\end{cases}
\]

where the wave number \( \kappa \) is given by

\[
\kappa = \sqrt{\frac{8k}{EH}} 
\]

Only the first half of this problem, Eq. (10a), needs to be solved to find an expression for \( v_0 \).

Assuming that the adherends deform elastically according to the Euler-Bernoulli beam theory and by prescribing the normal forces and the bending moments acting at \( x = 0 \) give the boundary condition, cf. Alfredsson (2004),

\[
v'(0) = -\frac{8\tau a}{EH}.
\]

A second boundary condition is given by assuming that the adhesive deformation is zero at the point of load application, \( x = b \). Thus,

\[
v(b) = 0.
\]

This is an approximation which is justified for the cases where the distance between the crack tip and the loading point is long enough. Using Eqs. (12) and (13) with the general solution Eq. (10a) gives the integration constants \( A_1 \) and \( A_2 \). If \( \kappa b \) is large enough these can be simplified to

\[
\tau v \sim J_{ct}
\]

**Figure 6** The constitutive relation for a linear elastic adhesive.
Numerical simulations show that this is a good approximation as long as $\kappa b > 5$, cf. Alfredsson (2004). Inserting the constants, $A_1$ and $A_2$, in Eq. (10a) and evaluating for $v(0) = v_0$ gives

$$v_0 = \frac{\bar{v}}{k} (\kappa a + 1). \quad (15)$$

Using this approximation in Eq. (9) yields,

$$J_{cl} = J_0 \left(1 + \frac{1}{\kappa a}\right)^2 \quad (16)$$

where $J_0$ is the energy release rate for a rigid adhesive layer, cf. Eq. (8). Thus, the difference between the fracture energy for a linear elastic adhesive and a rigid adhesive is a correction factor that generally gives higher values for a flexible adhesive layer than for a rigid. If the fracture energy is evaluated with only the term $J_0$, a too small value of the fracture energy is obtained. The underestimation of $J$ increases with the flexibility of the adhesive layer.

### 2.3 A general adhesive layer

For a general adhesive layer the force-displacement relation cannot be obtained analytically. However, an expression for the energy release rate can be derived. A derivation based on the $J$-integral is presented here. An alternative method to derive the energy release rate is given by Alfredsson (2004).

\[ A_1 = -\frac{r}{k} e^{-\kappa b}, \quad A_2 = \frac{r \kappa a}{k} \quad (14a, b) \]

Figure 7 The closed path.
The $J$-integral, Eq. (1), is evaluated for the path ABCD according to Fig. 7. The horizontal curves, A and C, are close to the adherend/adhesive interface. The two vertical curves go close to the crack tip (B) and just to the left of the loading point (D), respectively. An evaluation of $J$ along the closed path gives,

$$J = J_A + J_B + J_C + J_D = 0$$  \hspace{1cm} (17)$$

where $J_B$ and $J_D$ arise due to the strain energy density in the adhesive layer. Very small contributions from the stress tensor components appear at B and D and are therefore neglected here. The simplifications made will be discussed in Section 7.2. The terms $J_B$ and $J_D$ are given by

$$J_B = \int_0^v W_s \, dy = -\int_0^{v_0} \tau(v) \, dv = -J_{ad}$$  \hspace{1cm} (18)$$

$$J_D = \int_0^v W_s \, dy = \int_0^{v(b)} \tau(v) \, dv.$$  \hspace{1cm} (19)$$

For the two horizontal curves along the specimen, A and C, there is no contribution to the $J$-integral from the strain energy density, $W_s$, since $dy = 0$. The only contribution originates from the traction vector $T_i = \pm \tau_{xy}$ where the sign depends on the lower/upper adherend. Since the shear stress is continuous across the beam, the shear stress in the adherend close to the adhesive interface equals the shear stress, $\tau$, in the adhesive. With the coordinate system in Fig. 8, the component of the traction vector, $T_i = -\tau$ for the upper adherend and $T_i = +\tau$ for the lower adherend. Thus, along A and C,

$$J_A = \int_{-T_i u_{i,x}} \, dS = \int_{A} \tau e_{xx}^u \, dS = \int_{0}^{b} \tau e_{xx}^u (-dx) = \int_{0}^{b} \tau e_{xx}^u \, dx$$  \hspace{1cm} (20)$$

$$J_C = \int_{-T_i u_{i,x}} \, dS = \int_{C} -\tau e_{xx}^l \, dS = \int_{0}^{b} \tau e_{xx}^l \, dx$$  \hspace{1cm} (21)$$

where the contributions from the normal compressive stress at the loading point $\sigma_{yy}$ are neglected.

9
Thus,

\[ J_A + J_C = \int_0^b \tau \varepsilon_{xx}^u \, dx - \int_0^b \tau \varepsilon_{xx}^l \, dx = \int_0^b \tau (\varepsilon_{xx}^u - \varepsilon_{xx}^l) \, dx \]  

(22)

where \( \varepsilon_{xx}^u = \frac{d u^u}{dx} \) and \( \varepsilon_{xx}^l = \frac{d u^l}{dx} \). Here, \( u^u \) and \( u^l \) are the horizontal displacements at the adhesive/adherend interface of the upper/lower adherend, respectively, cf. Fig. 8.

Equation (22) can be rewritten as

\[ J_A + J_C = \int_0^b \frac{d}{dx}(u^u - u^l) \, dx = -\int_0^b \tau v' \, dx . \]  

(23)

where \( v = u^l - u^u \) is the relative longitudinal displacement of the adherend interfaces. It is identified as the shear deformation of the adhesive layer, cf. Fig. 8. Evaluating the \( J \)-integral for the curves A and C in Eq. (23) by inserting the expression for the shear stress from the differential equation Eq. (5a) gives

\[ J_A + J_C = -\int_0^b \frac{E H}{8} v''(x) v'(x) \, dx - \int_0^b \tau v'(x) \, dx \]  

(24)

The first and second terms in Eq. (24) are integrated,

\[ \int_0^b \frac{E H}{8} v''(x) v'(x) \, dx = \frac{E H}{8} \left[ \frac{v'(x)^2}{2} \right]_0^b = \frac{E H}{16} \left[ v'(b)^2 - v'(0)^2 \right] \]  

(25)
\[
\int_0^b \tau v'(x) \, dx = \frac{3}{8} \frac{P}{WH} \left[ v(b) - v(0) \right]. \tag{26}
\]

Introduction of \( v'(0) \) according to the boundary condition Eq. (12) in Eq. (25) gives with Eq. (26) the contribution from paths A and C. Inserting Eq.’s (18), (19) and (24) in Eq. (17) gives the total energy release rate for the adhesive layer,

\[
-J_B = J_A + J_C + J_D = \frac{9}{16} \frac{P^2 a^2}{E W^2 H^3} + \frac{3}{8} \frac{P}{WH} \left[ v(0) - v(b) \right] - \frac{EH}{16} v'(b)^2 + \int_0^b \tau(v) \, dv \tag{27}
\]

This equation is derived by Alfredsson (2004) based on beam theory. Equation (27) can be simplified to an expression with only measurable quantities. When the crack starts to propagate at \( x = 0 \), cf. Fig. 5, \( v(0) \) has reached the value at the end of the shear stress-shear deformation curve in Fig. 2. At the same time, at \( x = b \), the value of \( v(b) \) is close to zero if the distance \( b \) is long enough, cf. Fig. 9.

![Figure 9](image-url)

**Figure 9** The shear stress distribution at fracture along the adhesive layer for a linear elastic adhesive and for specimen N from simulations using experimental data.

Therefore, if the distance between the crack tip and the loading point is long enough \( v(0) \gg v(b) \) and \( v(b) \) can be neglected in Eq. (27). Equation (27) is then reduced to

\[
-J_B \approx \frac{9}{16} \frac{P^2 a^2}{E W^2 H^3} + \frac{3}{8} \frac{P}{WH} v(0) - \frac{EH}{16} v'(b)^2. \tag{28}
\]

Both the first and second term in Eq. (28) include only measurable quantities. However, the third term needs to be simplified further. In Fig. 9 the shear stress distribution at fracture along an ENF-specimen with a linear elastic adhesive layer without a crack, is compared to
simulation of an ENF-specimen from one of the experiments. The experimental curve is calculated with the finite element method using interface elements, cf. section 7.1. This simulation corresponds to the experiment with the longest process zone. It is reasonable to assume that this is the most critical case. At the loading point, \( x = b \), \( v'(b) \) for the linear elastic adhesive without a crack is approximately equal to the simulation result. For the linear elastic adhesive \( v'(b) \) is given by differentiating Eq. (10a). For \( a = 0 \) and assuming that \( \kappa b \) is large enough, this expression is simplified as

\[
v'(b) = -\frac{\tau}{k} \kappa = -\frac{P}{W} \sqrt{\frac{9}{8} \frac{1}{E k H^3}}
\]  

(29)

Inserting Eq. (29) into Eq. (28) gives an expression for the fracture energy that only includes measurable quantities.

\[
-J_B = \frac{9}{16} \frac{P^2 a^2}{E W^2 H^3} + \frac{3}{8} \frac{P v_0}{W H} - \frac{9}{128} \frac{P^2}{k W^2 H^2} = J_{ad}(v_0)
\]  

(30)

Thus, \( J_{ad} \) consists of three terms

\[
J_0 = \frac{9}{16} \frac{P^2 a^2}{E W^2 H^3}
\]  

(31a)

\[
J_1 = \frac{3}{8} \frac{P v_0}{W H}
\]  

(31b)

\[
J_2 = \frac{9}{128} \frac{P^2}{k W^2 H^2}
\]  

(31c)

and

\[
J_{ad} = J_0 + J_1 - J_2
\]  

(32)

The only quantities that need to be measured in the experiment are the applied force, \( P \), and the shear deformation at the crack tip, \( v_0 \). The initial stiffness of the adhesive layer, \( k \), is also evaluated from the experiments, cf. Section 4.4.
Figure 10 shows the evaluation of the different terms in the $J_{ad}$-expression for the adhesive used in the present paper, cf. Section 4. It is seen that the first and second term in Eq. (32) gives a significant contribution to the energy release rate, $J_{ad}$. The term $J_0$ contributes about 75 % and $J_1$ about 25 % to the value of $J_{ad}$ at fracture. The last term, $J_2$, is very small compared to the other terms; it only contributes 0.1 % to the total value of $J_{ad}$. Thus, in some of the calculations to follow, the term $J_2$ is neglected.
3 SPECIMEN DESIGN

The conditions for the validity of Eq. (30) described above have to be considered in the design of the specimens. For Eq. (30) to be valid, the adherends have to deform elastically and the condition \( v(0) \gg v(b) \) has to be fulfilled. A stable crack growth is also important to capture the complete stress-deformation curve in the experiments. Moreover, the deflection of the specimen has to be small compared to the height of the specimen in order for the beam theory to be valid.

3.1 Stable crack propagation

For the crack propagation to be stable, the length of the crack has to be long enough. In Carlsson et al. (1986) and Chai and Mall (1988) it is shown that, for the case of a rigid adhesive layer, the critical value of the crack length is \( a \geq 0.35L \). A more detailed analysis is given in Alfredsson (2004), accounting for the flexibility of the adhesive layer. This shows that this condition overestimates the critical crack length with about ten percent for the present adhesive. In order to obtain a small specimen, still having a margin to instability, \( a = 0.35L \) is used here.

3.2 \( v(0) \gg v(b) \)

The maximum value of the crack length, \( a_{\text{max}} \), has to be determined so that the condition \( v(0) \gg v(b) \) is fulfilled. To do this the adhesive is described by a sawtooth model, cf. Fig. 11.

![Figure 11](image)

**Figure 11** The sawtooth model is a simplified constitutive relation for the adhesive layer (left). An estimate of the shear stress distribution along the adhesive layer is used in the design of the specimen. The stress distribution is compared to the stress distribution for the sawtooth model. The shear stress distribution for the sawtooth model is a result of a FE-simulation described in Section 7.1 (right).
The sawtooth model is a simplified constitutive relation for the adhesive layer used by e.g. Stigh (1988) and Alfredsson (2004). It is a piecewise linear model with an initial part with increasing stress followed by a linear softening part. At \( v = v_c \) the adhesive layer has lost its load bearing capacity and a crack starts to propagate. The fracture energy for the sawtooth model is

\[
J_c = \frac{\tau_a v_c}{2}.
\]  

(33)

Generally, the process zone, i.e. the zone with softening behaviour, increases in length until the crack starts to propagate. At this point \( v(b) \) is at its largest value and \( P = P_c \). The critical load \( P_c \) is calculated from Eq. (30) with \( J_{ad} = J_c \) and \( v_0 = v_c \). Neglecting the last term \( J_2 \), leads to the following relation between \( P \) and \( v_0 \),

\[
P(v_0) = \frac{EWH^2v_0}{3a^2} \left( \sqrt{1 + \frac{1}{\gamma^2}} - 1 \right)
\]

where \( \gamma = \frac{1}{4} \sqrt{\frac{EH}{J_{ad}(v_0)/a}} \)

(34)

With \( v_0 = v_c \), this equation gives \( P_c = P(v_c) \). If the condition \( v(0) > v(b) \) is fulfilled, then the shear stress at the loading point is approximately equal to zero, thus \( \tau(b) = 0 \) and an approximate shear stress distribution along the adhesive layer is given in Fig. 11. Longitudinal equilibrium for an adherend in the \( x \)-direction gives,

\[
\frac{\tau_s b}{2} = \tau_c \frac{L}{2}
\]

(35)

where \( \tau_c \) is \( \tau \) at \( P = P_c \) given by combining Eq. (6) and Eq. (34),

\[
\tau_c = \frac{EHv_c}{8a^2} \left( \sqrt{1 + \frac{1}{\gamma^2}} - 1 \right)
\]

(36)

Inserting the expressions for \( \tau_c \) from Eq. (36) and \( \tau_a \) from Eq. (33) into Eq. (35) and expressing \( b \) as \((L/2 - a)\), cf. Fig. 5, gives an equation for the maximum crack length, \( a_{max} \),

\[
2\gamma^2 \left( \sqrt{1 + \frac{1}{\gamma^2}} - 1 \right) - \left( 1 - \frac{2a_{max}}{L} \right) = 0
\]

(37)

Where it should be noted that \( \gamma \) is a function of \( a_{max} \), cf. Eq. (34). This gives the maximum crack length, \( a_{max} \), at a specific length, \( L \), for different heights of the specimen.
3.3 Elastic adherends

The dimensions of the specimen have to be large enough to avoid plastic deformation of the adherends. Beam theory is used to estimate the maximum stress. Here, it is assumed that the adhesive fractures at the force, \( P = P_c \), which is a good approximation to the maximum force, \( P_{\text{f}} \). Fig 23. The force \( P_c \) is given by Eq. (34) as described above. The maximum stress in the adherends occurs at the crack tip and is given by

\[
\sigma_c = \frac{E v_c}{2a} \left( \sqrt{1 + \frac{1}{\gamma^2}} - 1 \right)
\]

(38)

Values of \( J_c \) and \( v_c \) are taken from the first experiment.

3.4 Small deformations

The inverse formula is based on linearised beam theory. A basic assumption is that the deflection is small as compared to the dimensions. Thus, \( \Delta \) should be less than \( H \). This condition is checked at the end of the design of the specimen using finite element analysis (ABAQUS). The stress-deformation relation is taken from the first experiment.

3.5 Results

The choice of the heights of the specimen is also limited by the dimensions that can be delivered by the manufacturer. The plasticity condition, Eq. (38), shows that the height, \( H \), must be larger than 7 mm if the crack length, \( a \), is chosen to 350 mm. The deflection of the specimen with \( H = 7 \) mm is 31 mm and it is 8.4 mm with \( H = 16 \) mm. The heights are therefore chosen to \( H = [16, 25, 32] \) mm. For \( v(0) \gg v(b) \), Eq. (37) give \( a_{\text{max}} = [408, 381, 362] \) mm, for \( H = [16, 25, 32] \) mm. To fulfil this requirement for all specimens and to simplify the manufacturing of the specimens, the crack length, \( a \), is 350 mm for all specimens. The length of all specimens, \( L \), is 1.00 m, \textit{i.e.} the stability condition in section 3.1 is fulfilled.
4 EXPERIMENTS

As described above, experiments are performed to evaluate the fracture energy, $J_c$, and the $\tau-v_0$-curve of the adhesive layer. In the experiments the applied force, $P$, and the shear deformation at the crack tip, $v_0$, are to be measured to evaluate, $J_{ad}(v_0)$, cf. Eq. (30).

The adhesive, DOW Betamate XW1044-3, is a toughened epoxy. A SEM-study shows that the adhesive consists of two substances; an epoxy/thermoplastic blend and a mineral compound, cf. Salomonsson and Andersson (2005). The mineral occupies about 25 % of the volume and appears in clusters surrounded by the polymer blend. The adherends are made of tool steel (Rigor Uddeholm) with a yield strength $\sigma_y > 500$ MPa. Young’s modulus, $E$, enters Eq. (30) and it is important to use the correct value. To this end, three three-point bending tests are used to measure Young’s modulus. All specimens are 1.00 m between the supports, the width is 32 mm and the heights are 16, 25 and 32 mm. About 10 different load levels are tested for each specimen. The average value of Young’s modulus is $E = 216$ GPa.

Experiments to determine $J_{ad}(v_0)$ are performed with four different dimensions of the specimens, cf. Table 1. In all experiments, the force is applied centrally and in all but one of the experiments the length of the crack, $a$, is 350 mm. In the first experiment performed, the crack length is 200 mm. This resulted in unstable crack propagation. Therefore, the crack length was extended to 350 mm in the rest of the experiments.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$L$ (m)</th>
<th>$a$ (m)</th>
<th>$H$ (mm)</th>
<th>$W$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td>0.20</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>B, N-O</td>
<td>1.00</td>
<td>0.35</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>C-D, K-M</td>
<td>1.00</td>
<td>0.35</td>
<td>25</td>
<td>32</td>
</tr>
<tr>
<td>E-J</td>
<td>1.00</td>
<td>0.35</td>
<td>16</td>
<td>32</td>
</tr>
</tbody>
</table>

Table 1 Specimen dimensions (nominal values)

It should be noted that each specimen is measured individually, and the measured values are used in the evaluation of the experiments.
4.1 Specimen preparation

The bonding surfaces of the adherends are cleaned with n-Heptane and rinsed in Acetone. To decrease the viscosity of the adhesive, both the adhesive and the adherends are preheated to about 60°C before the adhesive is applied to the adherend. Good alignment of the two adherends is secured with the use of a fixture. To achieve the correct adhesive thickness along the entire specimen, two steel wires with a diameter of 0.2 mm are adhesively bonded to one of the adherends, cf. Fig 12.

![Figure 12](image)

Figure 12 Steel wires are adhesively bonded to one of the adherends to achieve a layer thickness of the adhesive of 0.2 mm along the whole specimen. The steel wires start 100 mm from the crack tip.

According to simulations the process zone occupies less than 80 mm ahead of the crack tip, cf. Section 4.6. Hence, the steel wires start 100 mm from the crack tip to prevent interaction with the process zone. Moreover, the steel wires occupy only about 1 % of the cross sectional area of the adhesive. At the crack tip, a Teflon-film with a thickness of just over 0.2 mm is positioned. This ensures the thickness of the adhesive layer and also minimises the friction between the adherends during the experiment. To guarantee good adhesion between the adhesive and the adherends a small compressive force is applied to the specimens during the curing. This is done in an oven at 180°C for 30 minutes. After curing, the specimens are left in the oven to slowly cool to room temperature, thus, minimizing residual stresses in the adhesive layer. The cooling process takes about 12-16 hours depending on the size of the specimen.

4.2 Test procedure and experimental setup

The experiments are performed using a servo-hydraulic testing machine. The force, $P$, is measured with a load cell located between the actuator and the hydraulic grip. The displacement, $\Delta$, is measured with a Linear Voltage Differential Transformer (LVDT) positioned under the loading point. For the experimental setup, cf. Fig. 13.
The shear deformation at the start of the adhesive layer, $v_0$, is measured using an extensometer attached to two plates. These are fixed on each adherend on one side of the specimen. The extensometer is attached 7.5 mm above/below the crack tip, cf. Fig.'s 13 and 14. Figure 15 visualises that the extensometer measures the shear deformation.

The crack tip region is filmed during the experiments H-O using a CCD-camera attached to a microscope on the opposite side of the specimen as compared to the extensometer.
In experiments A-D, cf. Table 1, the displacement of the loading point is increased with a constant velocity of 1 mm/min. This leads to a considerably varying shear deformation rate during the test, from about 0.1 µm/s at the start of the experiment to about 10 µm/s at fracture, cf. Fig 16. In the other experiments, E-O, the velocity of the loading point is gradually decreased to obtain a constant velocity of the shear deformation at the crack tip, i.e., \( \dot{v}_o = 1 \) µm/s.

### Figure 15
The shear deformation is the relative displacement between the points where the plates are attached.

### 4.3 Fracture process

During the experiments the crack tip region is video recorded to study the deformation process of the adhesive. The deformation process in experiment L is analysed below and compared to the stress-deformation relation for the adhesive layer, cf. Fig. 17. The fracture process in experiment L is considered as typical for most of the experiments. In one of the experiments (J), the crack apparently propagates along the interface. All other data for this
experiment are consistent with the major part of the experiments. The conclusion is that for experiment J, the apparent interface crack is only propagating at the free surface.

To correlate the deformation of the specimen to the video, the clock of the data acquisition system and the video recorder are started using a countdown procedure. This gives a maximal error of about 1 s. This corresponds to an error of $1 \mu m$ in $v_0$.

It is observed that the microscopic fracture process usually starts some distance ahead of the crack tip. After initiating a number of slanted microscopic cracks, the cracks grow and coalesce to a long crack, cf. Fig 18.

![Figure 17](image)
The micro-crack initiation in the vicinity of the crack tip. The fracture process is shown at three different points on the stress-deformation curve, from experiment L.

![Figure 18](image)
After fracture the microscopic cracks grow and coalesce to a long crack.
In all but one of the experiments (J) the micro cracks are slanted *cf.* Fig. 17. Contrary to the results of Chai (1993) and others, the cracks do not appear to open in the direction of the maximum normal stress in the layer, *i.e.* 45°, *cf.* Fig. 20a. It is tempting to assume that this is a result of an additional compressive loading in the vertical direction in the ENF-specimen. However, numerical analyses show that the normal stresses are virtually zero at the crack tip, *cf.* Fig 19.

![Figure 19](image)

*Figure 19* The normal stress in the ENF-specimen at just before crack propagation starts, from experiment N.

Moreover, a compressive vertical stress leads to a principal stress direction corresponding to cracks with steeper slope, *cf.* Fig. 20b. Another possible reason for the unexpected slope of the cracks is effects of residual stresses in the horizontal direction. Due to different thermal expansion coefficients for the adhesive and the steel, residual stresses are likely to appear although the cooling process is very slow, *cf.* Section 4.1. If the steel has a lower thermal expansion coefficient than the adhesive, the cooling leads to a larger shrinkage of the adhesive than the steel adherends. Thus, a tensile horizontal stress appears in the adhesive layer. This would give cracks with a steeper slope than 45°, *cf.* Fig. 20c. This is not consistent with the results in the video. Compressive residual stresses would give cracks consistent with the videos, *cf.* Fig 20d. These are not consistent with a larger thermal expansion coefficient in the adhesive than in the adherends. However, the reports of 45°’s cracks appear to be coupled to much more brittle adhesives than the present one.
In all experiments, the adhesive layer experiences some peel deformation. This is apparent in Fig.'s 17 and 18. Simulations show small compressive stress in the vertical direction. Thus, the peel deformation is not attributed to secondary peel stresses. In the videos, it appears as if slanted parts of the adhesive work as the pole of a pole-vaulter and by local rotation the pole separates the adherends in the vertical direction. The mechanism is visualised in Fig. 21.

Figure 20  The state of stress for the adhesive layer at the crack tip in four different cases and the resulting principal stress directions. (a) Pure shear stress in the adhesive layer. (b) Shear and compressive stresses in the adhesive layer. (c) Shear and residual stresses in tension. (d) Shear and residual stresses in compression.
4.4 Evaluation of the experiments

Equation (30) gives the $J_{ad-v0}$-curve for an experiment. The third term in Eq. (30) contains the elastic stiffness of the adhesive layer, $k$, which has to be estimated from the experiments before Eq. (30) can be used. The elastic stiffness is evaluated from the initial part of the experimental $P(v_0)$-curve. This can be achieved by noting that for a linear elastic adhesive layer the shear deformation at the crack tip is given by Eq. (15). The initial gradient of the $P(v_0)$-curve, $c$, for a case of a linear elastic adhesive follows by use of Eq. (6),

$$c = \frac{P}{v_0} = \frac{8 \ W H k}{3 \ k a + 1} \quad \text{(39)}$$

With $\kappa$ from Eq. (11), the elastic stiffness is solved from Eq. (39),

$$k = \frac{9 a^2}{32 E H^3} \left(\frac{c}{W}\right)^2 \left(1 + \sqrt{1 + \xi}\right)^2, \quad \text{where} \quad \xi = \frac{4 E H^2 W}{3 a^2 c}. \quad \text{(40 a, b)}$$

The first 21 points measured in the experiments are used to evaluate the initial gradient, $c$. These points are approximated with a fourth order polynomial. The polynomial is differentiated and the gradient at the first point is the constant $c$. In Fig. 22 the procedure is shown. The estimated values of $k$ are given in Table 2.
The evaluation of the energy release rate, $J_{ad}$, involves differentiation of experimental data that will lead to scatter in the $\tau(v_0)$-curve. In order to minimise this, the $J_{ad}$-curve is first approximated with a Prony-series. This series is well established and has proven well suited to describe the $J_{ad}(v_0)$-relation.

$$J_{ad}(v_0) = \frac{J_c}{\tau_c} = \sum_{i=1}^n A_i \exp \left( -\frac{n}{v_c} \right).$$  \hspace{1cm} (41)

where $n = 10$ is used in the evaluation. A least square fitting procedure is used to determine the parameters $A_i$. From the videos it is hard to identify when the crack starts to propagate, cf. Fig. 17, since the adhesive layer can carry load even after the first small cracks have started to appear in the adhesive. Therefore, it is assumed that the crack starts to propagate at the maximum value of the energy release rate, $J_c$. The critical shear deformation $v_c$ is taken at this point, cf. Fig. 23.

Figure 22  The straight line is the approximated value of the parameter c in Eq. (39), from experiment J.

The crack propagation is assumed to start at the maximum value of the energy release rate (left). The crack propagation usually does not start at the maximum value of the force as shown in the figure (right).
Only the part of the curve up to the point \( v_c \) is considered in the following evaluation. As shown in Fig. 23, this is not necessarily at the maximum value of the force, \( P \). Thus, \( P \) is decreasing just before crack propagation. This result appears to be general; the crack starts to propagate after the force has reached its maximum value.

Some constraints are introduced in the least square approximation for the curve to follow a likely behaviour. Four constraints are introduced. These correspond to the shear stress equal to zero and the slope equal to the initial stiffness of the adhesive layer at no shear deformation. At fracture the shear stress is set equal to zero and the energy release rate is equal to the fracture energy, \( J_c \). After the least square fitting procedure the series is differentiated. The evaluation process do not give a large difference if the constraints on the \( J_{ad}(v_0) \)-curve are not included. One difference is that the \( \tau(v_0) \)-curve might not start at the origin which is physically unrealistic. Since the curve is broken off at the maximum value of \( J_{ad} \) and then approximated with the series in Eq. (41) this leads to \( \tau \neq 0 \) at the maximum shear deformation and \( J_{ad} \) may vary a little from \( J_c \) at this point. If the constraints are not introduced, the values of \( \tau_{max} \) and \( J_c \) varies less than 1 % as compared to the curves with constraints. The parameters that are evaluated from the experiments are the maximum shear stress, \( \tau_{max} \), the value of the shear deformation at the maximum shear stress, \( v_a \), the fracture energy, \( J_c \), the critical shear deformation, \( v_c \), and the elastic stiffness of the adhesive, \( k \). These parameters characterise the constitutive relation shown in Fig. 24.

![Figure 24](image.png)

**Figure 24** The parameters that characterise the constitutive relation for an adhesive layer loaded in shear. These parameters are the maximum shear stress, \( \tau_{max} \), the value of the shear deformation at the maximum shear stress, \( v_a \), the fracture energy, \( J_c \), the critical shear deformation, \( v_c \), and the elastic stiffness of the adhesive, \( k \). These parameters are evaluated from the experiments.
4.5 Experimental results

Experiments have been performed at four different occasions. Three different batches of the adhesive were used in experiments A-D, E-F and H-O, respectively. In experiments A-D the displacement rate of the loading point was held constant and in experiments E-O the shear deformation rate was held constant. In Fig. 25 the \( \tau - v_0 \)-curves from all experiments are presented. In Fig. 25f a comparison is made between the results from the different geometries and batches. Good repeatability is achieved in the experiments but some deviation is observed. The repeated experiments using the same batch of adhesive and the same geometry show excellent agreement with the exception of the experiments with \( H = 32 \) mm. No specific difference between the fracture surface of this experiment and the others is observed. Excluding the results from experiments B and D from Fig. 25f show no significant difference due to different geometries of the specimens. Thus, the \( \tau - v_0 \)-relation appears to be a constitutive property of the adhesive layer. However, there is a significant difference between the results from the experiments with a prescribed deflection rate and prescribed shear deformation rate.
Figure 25  $\tau$-$v_0$-curves for the fifteen ENF-tests. Figure (a) shows the four experiments performed under constant displacement rate of the loading point. Figure (b) corresponds to beam height 16 mm. Figures (c), (d), and (e) correspond to beam heights 16, 25 and 32 mm, respectively. Figure (f) shows a comparison of the results from different geometries and different batches.
Table 2 gives the evaluated parameters from the experiments. Note that experiment A gave an unstable crack growth. Hence, the values of $v_c$ is uncertain for this experiment. Figure 26 shows the fracture energy as a function of the height of the specimens. The figure shows that the four experiments in Group I and II have substantially larger fracture energy than the rest of the experiments. The reason for experiments A-D to give higher values might be that the shear deformation rate varies during these experiments while the value is constant in the rest of the experiments, cf. Fig. 16. Thus, the deformation rate is probably a parameter of the $\tau-v_c$ relation and its influence should be more carefully investigated. Supporting this conclusion, Sørensen (2002, 2003) shows that the fracture energy for adhesive layers loaded in peel depends on the loading rate. A higher loading rate is shown to result in a higher value of the
fracture energy which is consistent with the present results. However, Sørensen use the elastomer polyurethane which is expected to be more viscous than the thermoset epoxy used here. No significant batch to batch variation can be detected from the results in Table 2 and Fig. 26.

![Figure 26](image_url)

**Figure 26** Values of the fracture energy (left) and the maximum shear stress (right) for different heights of the specimen.

The elastic stiffness of the adhesive layer is expected to be rather constant and only depend on the thickness of the layer $t$. As shown in Table 2, $k$ varies substantially between the experiments; the smallest value is less than half the value of the largest. The value of $k$ enters the expression for the evaluation of $J_{ad}$, Eq. (30), in the third term. This term is shown to have a minute influence on $J_{ad}$, cf. Fig. 10. However, $k$ influences the length of the process zone, $p$. This is discussed in Section 4.6. It is obvious that the experimental method works poorly in predicting the elastic stiffness, $k$, of the adhesive. Depending on how many points that are used in the evaluation, the value of $k$ differs a lot.

Another method to determine $k$ is to use the relation between the shear modulus $G$ of the adhesive and the adhesive layer thickness $t$, given by

$$k = \frac{G}{t}. \quad (42)$$

The shear modulus is given by,

$$G = \frac{E_{ad}}{2(1 + \nu_{ad})}, \quad (43)$$

where $E_{ad}$ and $\nu_{ad}$ are Young’s modulus and Poisson’s ratio of the adhesive, respectively. Bulk tests on the adhesive give $E_{ad} = 2$ GPa and $\nu_{ad} = 0.4$. Using the measured value of the adhesive
layer thickness, \( t \), for each specimen, cf. Table 2, a new value for the elastic stiffness, \( k^* \), is calculated. The data are given in Table 2. Using this value of \( k \) gives an alternative value of \( \tau_{\text{max}} \) given in the tenth column of Table 2. As expected, the results are almost the same as with the original values of \( k \).

4.6 Length of the process zone

By studying the length of the process zone, \( p \), it can be determined if the distance, \( b \), between the crack tip and the loading point is large enough so that the simplifications of Eq. (27) are justified. The first simplification is to neglect \( v(b) \) as compared to \( v(0) \) and the second is to approximate \( v'(b) \) with the corresponding value for an elastic adhesive.

The process zone is defined as the part of the adhesive layer in front of crack tip where the adhesive softens, i.e. where \( v_0 > v_a \) cf. Fig. 24. The length of the process zone, \( p \), can be determined by studying the shear stress distribution along the ENF-specimen from simulations, cf. Fig. 27.

![Figure 27](image-url)  
*Figure 27* The length of the process zone is evaluated from the stress distribution along the adhesive layer. Finite element simulations are used to determine the shear stress distribution along the adhesive layer for each specimen by inserting the constitutive relations from the experiments.
Finite element simulations, cf. Section 7.1, are used to determine the shear stress distribution along the adhesive layer. In each simulation, the constitutive relation from that experiment is used in the simulation.

The evaluated length at the moment of crack initiation is given in Table 2. It is shown that the length depends on the height of the adherend. In all experiments, the process zone is substantially larger than the thickness of the adhesive layer. Therefore linear elastic fracture mechanics is not expected to be applicable according to the ASTM-condition, cf. Section 5.2.

The values of $p$ in Table 2 are estimates of the length of the process zone. In Fig. 28, $p$ is given as a function of $H$ for all specimens. As shown in the figure, the same height of the specimen gives different values of $p$. This is probably due to the fact that the length of the process zone strongly depends on the elastic stiffness of the adhesive layer. This value varies substantially between different experiments.

![Figure 28](image)

**Figure 28** The length of the process zone, $p$, for different heights of the specimen.

The results in Fig. 28 show a tendency that higher adherends give a longer process zone. The length of the process zone is always less than 80 mm, cf. Table 2. This value should be compared to $b = 150$ mm. Thus, for all specimens $p$ is substantially shorter than $b$, cf. Fig. 27. This validates the use of the simplifications in Eq. (30). An estimate of the error due to the length of the process zone is presented in Section 7.1.
5 COMPARISON WITH LEFM

To examine the validity of the experimental results a comparison with load-deflection-curves based on a linear elastic fracture mechanics approach is performed. The force versus the deflection is compared. The analysis is performed with two simplified models of the adhesive. In the first case, the adhesive is considered to be rigid; in the second, the adhesive is considered to be linear elastic. The curves are divided into two parts; before the crack starts to propagate and during crack propagation. During the first part, the structure is linear elastic and the \( P-\Delta \)-curve is a straight line. In the second part, the energy release rate is constant and equal to \( J_c \).

At the critical force where the crack starts to propagate, the crack length, \( a \), is equal to the initial crack length, \( i.e. \) 350 mm. In the second part of the curve, \( a \) increases from the initial length to 480 mm.

5.1 Rigid adhesive

In Section 2.1, the energy release rate, \( J_0 \), is derived for a rigid adhesive layer, \( cf. \) Eq. (8). If the adhesive is assumed to fracture at \( J_0 = J_c \) the applied force has to vary to keep \( J_0 \) constant. Equation (8) gives,

\[
P(a) = \frac{4W}{3a} \sqrt{EH^3J_c}.
\] (44)

The deflection, \( \Delta \), is conveniently calculated using Castigliano’s theorem \( \Delta = \frac{\partial \bar{U}}{\partial P} \) where \( \bar{U} \) is the complementary energy of the specimen. In the case of a prescribed load and a linear elastic material, this energy equals the negative of the potential energy given in Eq. (7). Differentiating with respect to the force, \( P \), gives,

\[
\Delta = \frac{P}{32} \frac{12a^3 + L^3}{EH^3}
\] (45)

With \( P(a) \) from Eq. (44), the deflection during crack propagation is given by,

\[
\Delta(a) = \frac{1}{24} \left( \frac{12a^3 + L^3}{a} \right) \sqrt{\frac{J_c}{EH^3}}
\] (46)
Thus, the $P$-$\Delta$-curve is given in a parametric form by $P(a)$ from Eq. (44) and $\Delta (a)$ from Eq. (46). The force versus deflection curve before and during crack propagation is given in Fig. 30.

### 5.2 Linear elastic adhesive

For linear elastic systems the energy release rate can be expressed as

$$J = \frac{P^2}{2W} \frac{\partial C}{\partial a}. \quad (47)$$

The compliance of the ENF-specimen with a linear elastic adhesive layer can be calculated using elementary beam theory. An alternative method is used here. Combining the approximate expression for the $J$-integral for a linear elastic adhesive layer, Eq. (16), with Eq. (47) and integrating with respect to $a$ gives the compliance $C$ for an ENF-specimen. The result is

$$C(a) = \frac{1}{8EWH^3} \left[ 3a^3 + 9 \left( \frac{a}{\kappa^2} + \frac{a^2}{\kappa} \right) \right] + B \quad (48)$$

where the integration constant $B$ remains to be determined. This constant is given by the compliance for an ENF-specimen without a crack, $a = 0$, which is easier to determine than the compliance for the specimen with $a \neq 0$. The first step is to calculate the deflection at the loading point, $w(L/2) = \Delta$, cf. Fig. 29.

**Figure 29** Forces on the ENF-specimen to the left of the loading point (left). Forces acting on each adherend (right).
Assuming that the adherends deform elastically, according to the Euler-Bernoulli beam theory and small rotations, the relation between the bending moment and the deflection of one of the adherends is given by

\[
M(x) = -\frac{EWH^3}{12} w''(x). \quad (49)
\]

where \(w(x)\) is the deflection of the beam and \(w''(x)\) the curvature. According to Fig. 29 the bending moment is given by,

\[
M(x) = \begin{cases} 
\frac{1}{4} P x - \frac{1}{2} HN(x) & 0 \leq x \leq L/2 \\
\frac{1}{4} P (L - x) - \frac{1}{2} HN(x) & L/2 \leq x \leq L 
\end{cases}. \quad (50)
\]

Longitudinal equilibrium of an adherend, cf. Fig. 29, gives the normal force,

\[
N(x) = W \int_0^x \tau[v(\tilde{x})] \, d\tilde{x} = Wk \int_0^\tilde{x} v(\tilde{x}) \, d\tilde{x} \quad (51)
\]

where the shear stress for a linear elastic adhesive, \(\tau = kv\), is used in the last expression. The shear deformation, \(v\), is given by Eq. (10a). Inserting in Eq. (51) gives an expression for the normal force to be used in Eq. (50). The deflection, \(w(x)\), is calculated by integrating Eq. (49) twice. The two integration constants are given by the two boundary conditions \(w(0) = 0\) and \(w'(L/2) = 0\) and the deflection of an ENF-specimen without a crack, \(a = 0\), is \(\Delta = w(L/2)\). From this expression, the compliance, \(C = \Delta/P\), is given as

\[
C(a = 0) = \frac{1}{32EWH^3} \left( L^3 - \frac{72}{k^3} + 36 \frac{L}{k^2} \right) \quad (52)
\]

With Eq. (48), the integration constant \(B\) is thus determined. Inserting in Eq. (48) gives the compliance for the ENF-specimen with a crack

\[
C(a) = \frac{1}{32EWH^3} \left[ 12a^3 + L^3 + 36 \left( \frac{a}{k^2} + \frac{a^2}{k} + \frac{L}{k^2} \right) - \frac{72}{k^3} \right] \quad (53)
\]

An alternative derivation of this equation is given by Alfredsson (2004). With \(a\) and \(P\) determined, Eq. (53) gives the corresponding \(\Delta\).
The force $P$ corresponding to a growing crack is given by using $J_0$ from Eq. (8) in the expression for $J_{el}$ of Eq. (16). With $J_{el} = J_c$, the force $P$ can be expressed as a function of $a$,

$$P(a) = \frac{4W \sqrt{EH^3 J_c}}{3a(1+1/\kappa a)}.$$  \hspace{1cm} (54)

Figure 30 shows the force versus the deflection of the loading point from experiments J, L and N. It is expected that the experimental and the simulated $P$-$\Delta$-curve for an elastic adhesive shall coincide as long as the adhesive layer behaves elastically. This appears to be the case for $\Delta < 1$ mm for experiment K and L. A small deviation is shown for experiment N. However, for $\Delta > 1$ mm in Fig. 30, the experimental curves show somewhat more flexibility than the linear elastic one. As expected, the rigid adhesive layer gives a somewhat stiffer response than the elastic and the experimental behaviour. The critical force is generally larger than the experimental values. This is expected from Eq. (30) since the inclusion of flexibility of the adhesive yields larger values of $J_{ad}$. The critical force as calculated using a rigid adhesive model is less than 16 % in error as compared to the experiment.

Figure 30  Load vs. deflection of the loading point for three experiments; Experiment J (upper left corner), experiment L (upper right corner) and experiment N (lower).
The analysis performed above clearly shows that the experimental results are reasonable. The experimental $P-\Delta$ curves are beneath the theoretical curves, as they should.

LEFM works surprisingly well for predicting the critical load. This indicates that the ASTM-condition is very conservative for this application. The condition states that all length parameters shall fulfil

$$l > 2.5 \left( \frac{K_{lc}}{\sigma_y} \right)^2 = 2.5 \frac{G_c E'}{\sigma_y^2}$$

(55)

where $K_{lc}$, $\sigma_y$ and $G_c$ are the fracture toughness, the yield strength, and the fracture energy of the adhesive, respectively. Moreover, $E' = E_{ad}$ for plane stress and $E' = \frac{E_{ad}}{1 - \nu_{ad}^2}$ for plane strain. In this case by approximating $G_c$ with $J_c$ from experiment J, with the yield strength of the adhesive, $\sigma_y = 35$ MPa, and assuming plane strain conditions gives a critical length parameter of 11 mm. Thus, all length should be larger than 11 mm. The condition is clearly violated in this case since the thickness of the adhesive layer is less than 11 mm.
6 ERROR ESTIMATES

Three different error analyses are performed to investigate the accuracy of the measurements and the evaluation process. This is important since the procedure described above requires a differentiation of experimental data. Thus, the demand on the accuracy of the test equipment is high. First, an analysis of the influence of randomised errors in the data acquisition equipment is performed. In the second investigation, the influence of systematic errors in the data acquisition equipment is analysed. Lastly, errors due to the dimensions and material parameters are investigated. Errors in the fracture energy, $J_c$, the maximum value of the shear stress, $\tau_{\text{max}}$, and the initial stiffness, $k$, are evaluated.

6.1 Monte-Carlo simulation

Monte-Carlo simulations, cf. e.g. Rubinstein (1981) and Andersson and Stigh (2004), are performed to evaluate how randomised errors in the measured values of the applied force and the shear deformation influence the evaluation process.

An artificial $\tau-v_0$-curve is designed to approximate a $\tau-v_0$-curve from the experiments, cf. Fig 31. The $\tau-v_0$-curve from experiment J is reproduced by using a combination of three different curves in three intervals. The first part, from the origin to (a) in Fig 31, is linear with a slope that equals the initial stiffness, $k$, of the adhesive. A Prony-series with 6 terms, cf. Eq. (41), is used in the second interval from (a) to (c). In this interval, the curve is based on conditions at three points, cf. Fig 31. In point (a) the slope equals the initial stiffness of the adhesive, the peak value at (b) equals the maximum value of $\tau$ from the experiment and at point (c) the slope is chosen to achieve a correct value of $J_c$.

![Figure 31](image_url)

The theoretical curve and the experimental curve are plotted on the left graph. The corresponding $J_{\text{ad}}-v_0$ curves are shown on the right graph.

Figure 31 The artificial $\tau-v_0$-curve (solid line) designed to approximate the $\tau-v_0$-curve from the experiment (dotted line) (left). The corresponding $J_{\text{ad}}-v_0$-curves (right).
In the design of the artificial $\tau$-$v_0$-curve the following input data is used; the fracture energy, $J_c = 2.24$ kJ/m$^2$, the maximum shear stress, $\tau_{\text{max}} = 25.9$ MPa and the initial stiffness, $k = 2990$ GN/m$^3$. The input data comes from experiment J. Integration of the $\tau$-$v_0$-curve according to Eq. (2) yields the $J_{\text{ad}}$-$v_0$-curve, cf. Fig. 31. The $P$-$v_0$-curve is given by solving Eq. (30) and neglecting the last term which has been shown to give a very small contribution, cf. Fig. 10. The solution is given by Eq. (34). This gives the $P$-$v_0$-curve corresponding to an idealised case where all data are measured without errors.

Randomised errors are applied to the force, $P$, and the shear deformation, $v_0$. A normal distribution is assumed. The shear deformation is simulated to be measured at equidistant intervals giving a series of measured values of $v_m$. To every value, $v_m$, a randomised error, $E_v$, is added mimicking the errors in the data acquisition instruments. This gives a new value of the shear deformation, $v_t$, cf. Fig. 32 which corresponds to the correct value of $v_0$. The value of the force, $P(v_t)$, at this point is calculated with Eq. (34). An additional error, $E_p$, is then added to the $P(v_t)$-value simulating errors in the force measurement, cf. Fig. 32. The new value is denoted $P_m$ and simulates the measured value at $v_m$. In this way a series of simulated measurement values ($v_m$, $P_m$) is obtained.

![Figure 32](image-url)  
**Figure 32** Schematic figure showing how the errors are generated in the Monte-Carlo simulation.
The size of the randomised errors is estimated from the calibration curves for the test equipment. The calibration error is different for different readings in the test equipment but it is always given as a percentage of the full scale range. In all experiments the full scale range of the load cell is 100 kN and for the extensometer it is 4 mm. The calibration error is 0.015 % of the full scale range for the extensometer and less than 0.001 % for the load cell, cf. Appendix. With $e$ denoting the error, a randomised error is obtained using the Matlab function `randn` which generates normally distributed numbers. The standard deviation is set to $e/2.576$ and the average value is set to zero. The number 2.576 corresponds to 99 % of all cases if a normal distribution is assumed. This number is used throughout this investigation to couple a ± value with the standard deviation. The randomised numbers are added to the simulated data as described above. Two analyses are first performed. In case (I), the randomised errors are assumed to be equal in size to the calibration errors. In case (II), the errors are assumed to be two times larger.

When the errors are added a new $J_{ad}-v_0$-curve is calculated using Eq. (30). The $\tau$-$v_0$-curve is calculated using the procedure described in Section 4.4. This $\tau$-$v_0$-curve is compared to the theoretical $\tau$-$v_0$-curve, cf. Fig 33. The dotted curve is the one from the simulation.

![Figure 33](image) The artificial $\tau$ ($v_0$)- and $J_{ad}$($v_0$)-curve and the corresponding curves with added errors for the case (I).

For each generated curve, the parameters $J_c$, $k$ and $\tau_{\text{max}}$ are evaluated. A total of 2000 simulations are performed in the Monte-Carlo simulation.

Normal probability plots and histograms for the parameters $J_c$, $k$ and $\tau_{\text{max}}$ for case (I) are shown in Fig 34. The vertical lines in the histograms show the input data for $J_c$ and $\tau_{\text{max}}$. The input data for $k$ diverge too much and is therefore not visible in the histogram. Figure 34,
shows that $J_c$ and $k$ appears normally distributed. This is not the case for $\tau_{\text{max}}$. The average value for $J_c$ is 2.24 kJ/m$^2$ and the standard deviation is 0.72 J/m$^2$. With this accuracy, the average value equals the input data. For the initial stiffness, $k$, the average value is 3.21 TN/m$^3$ and the standard deviation is 24 GN/m$^3$. The average value overestimates the original value with about 7 %. The maximum shear stress, $\tau_{\text{max}}$, does not fit any known distribution. However, the average value is 26.2 MPa and only overestimates the input data of $\tau_{\text{max}}$ with 1.2 %. The standard deviation is 0.21 MPa.

Figure 34  Normal probability plots and histograms of $J_c$, $\tau_{\text{max}}$ and $k$ for case (I). The vertical lines in the histograms show the input data for $J_c$ and $\tau_{\text{max}}$. The input data for $k$ diverge too much and it is therefore not visible in the histogram.
Figure 35 shows the results from case (II). The fracture energy, $J_c$, still shows a normal distribution. The standard deviation is $1.4 \text{ J/m}^2$. This is twice the standard deviation in case (I). To the second decimal it gives the same average value of the fracture energy as the input data. Thus, randomised errors do not generate errors in the average value of $J_c$. A normal distribution is no longer achieved for the initial stiffness, $k$. The average value of $k$ overestimates the original value with about $8\%$. The standard deviation is $156 \text{ GN/m}^3$. Thus, the average value is only slightly inferior to case (I). However, the standard deviation increases more than six times. As for case (I), the maximum shear stress is not distributed as any known distribution. However, the average value only overestimates the original value of $\tau_{\text{max}}$ with $1.2\%$ which is the same as for case (I). The standard deviation is $0.23 \text{ MPa}$, almost the same as in case (I). The conclusion is that a doubling of the randomised errors gives small changes to the accuracy of the measured parameters. The exception is the initial stiffness which gets a significantly larger standard deviation with the larger error.
The results from cases (I) and (II) do not indicate that the procedure would give as large scatter in $k$ as found in the experiments, cf. Table 2. Some singular values of $k$, after 2000 simulations for case (II), show greatly deviating values, cf. Fig. 35. The large scatter in $k$ found in the experiments, cannot be explained by the randomised errors simulated here.

Another method can be developed based on the assumption that we have a good estimate of $k$ given by the bulk data of the adhesive, cf. Eq. (42). This value is then used in the evaluation of the simulated measured data. Therefore, case (III) is analysed with the elastic stiffness, $k$, 

Figure 35  Normal probability plots and histograms of $J_c$, $\tau_{\text{max}}$ and $k$ for case (II). The vertical lines in the histograms show the input data for $J_c$ and $\tau_{\text{max}}$. The input data for $k$ diverge too much and it is therefore not visible in the histogram.

The results from cases (I) and (II) do not indicate that the procedure would give as large scatter in $k$ as found in the experiments, cf. Table 2. Some singular values of $k$, after 2000 simulations for case (II), show greatly deviating values, cf. Fig. 35. The large scatter in $k$ found in the experiments, cannot be explained by the randomised errors simulated here.

Another method can be developed based on the assumption that we have a good estimate of $k$ given by the bulk data of the adhesive, cf. Eq. (42). This value is then used in the evaluation of the simulated measured data. Therefore, case (III) is analysed with the elastic stiffness, $k$, 

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held constant. Randomised errors as in case (II) are used for the force and shear deformation measurements.

Figure 36  Normal probability plots and histograms of $J_c$ and $\tau_{\text{max}}$ for case (III). The vertical lines in the histograms show the input data for $J_c$ and $\tau_{\text{max}}$.

Figure (36) shows the result for case (III). The average value of $J_c$ is still 2.24 kJ/m$^2$ and the standard deviation of 1.4 J/m$^2$ which is the same as for case (II). This is expected since the term in Eq. (30) influenced by $k$ only marginally influences the value of $J_{\text{ad}}$. The average value of $\tau_{\text{max}}$ is still 26.2 MPa, same as in case (II). The standard deviation is 0.22 MPa, also similar to the previous cases. Thus, the alternative procedure to determine $k$, viz. by a bulk test, does not provide better estimates of the parameters.

All data are given in Table 3.
6.2 Systematic errors

Systematic errors in the test equipment that measures the force, $P$, and the shear deformation, $v_0$, appear in the experiments. These are calibration errors. They are applied to the $P$ and $v_0$ values in the same way as described for the Monte-Carlo simulation. The calibration errors of the load cell and extensometer are added to obtain an estimate of the maximum possible systematic error. This procedure leaves $J_c$ practically unaffected. However, the maximum shear stress and the initial stiffness are overestimated with 0.4 % and 9 %, respectively. These results are shown in Table 3.

6.3 Errors in the dimension measurement and material data

Errors affecting the fracture energy due to errors in the dimensions and material data are estimated with the formula for propagation of error. Errors are added to the different dimensions of the specimen; $a$, $W$ and $H$ and also to the Young’s modulus, $E$. The width and the height of the specimen are measured with a micrometer and the error is estimated to be less than 5 $\mu$m. The crack length is measured with a ruler and the crack length may also be affected by the alignment of the specimen at the experimental setup. This alignment is made by visual inspection. The maximum error is estimated to be less than 2 mm. The error in the $E$-modulus measurement is estimated to be less than 6 GPa. The value of the absolute error in the fracture energy is given by

$$ J_e = E_a \frac{\partial J}{\partial a} + E_w \frac{\partial J}{\partial W} + E_H \frac{\partial J}{\partial H} + E_E \frac{\partial J}{\partial E}. \quad (56) $$

This gives an error in the fracture energy, $J_e$, of $\pm 0.07$ kJ/m$^2$ which is about 3 % of $J_c$ for specimen J.

Misalignment of the specimen may also lead to an incorrect value of the force. This error is probably rather small due to the substantial length of the specimen. Since the error estimated in Eq. (56) is small, the error due to misalignment is judged to be even smaller.
6.4 Results

The values of the evaluated parameters $J_c$, $\tau_{\text{max}}$ and $k$ are summarised in Table 3. The error in each value is given by 2.576 times the standard deviation as described above.

Table 3 The results from the error estimation analysis for the parameters $J_c$, $\tau_{\text{max}}$ and $k$. The average values and the errors from each analysis are given.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$J_c$</th>
<th>$\tau_{\text{max}}$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input data</td>
<td>2.24 kJ/m$^2$</td>
<td>25.9 MPa</td>
<td>2990 GN/m$^3$</td>
</tr>
<tr>
<td>Monte-Carlo (Case I)</td>
<td>2.24 ± 2·10$^{-3}$ kJ/m$^2$</td>
<td>26.2 ±0.5 MPa</td>
<td>3210 ±60 GN/m$^3$</td>
</tr>
<tr>
<td>Deviation between input data and average value (%)</td>
<td>0 %</td>
<td>1.2 %</td>
<td>7.4 %</td>
</tr>
<tr>
<td>Monte-Carlo (Case II)</td>
<td>2.24 ±4·10$^{-3}$ kJ/m$^2$</td>
<td>26.2 ±0.6 MPa</td>
<td>3230 ±400 GN/m$^3$</td>
</tr>
<tr>
<td>Deviation between input data and average value (%)</td>
<td>0 %</td>
<td>1.2 %</td>
<td>8%</td>
</tr>
<tr>
<td>Monte-Carlo (Case III)</td>
<td>2.24 ± 4·10$^{-3}$ kJ/m$^2$</td>
<td>26.2 ±0.6 MPa</td>
<td>prescribed</td>
</tr>
<tr>
<td>Deviation between input data and average value (%)</td>
<td>3.1 %</td>
<td>1.2 %</td>
<td>-</td>
</tr>
<tr>
<td>Systematic</td>
<td>2.24 kJ/m$^2$</td>
<td>26.0 MPa</td>
<td>3260 GN/m$^3$</td>
</tr>
<tr>
<td>Deviation between input data and average value</td>
<td>0 %</td>
<td>0.38 %</td>
<td>9.0 %</td>
</tr>
<tr>
<td>Dimensions</td>
<td>2.24 ±0.07 kJ/m$^2$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The error analysis shows that errors of the same size as the calibration errors give small effects on the results. Neither randomised nor systematic errors give any substantial error. The initial stiffness, $k$, is more sensitive to errors in the measurements than the other parameters. The major contributors to the error in fracture energy are errors in the dimensions and Young’s modulus of the adherends.
7 VALIDATION OF THE METHOD

In Section 5 we compared the experimental results with predictions using LEFM. The correspondence was good. In order to check if the simplifications regarding the deformation of the adhesive under the loading point are justified and also to check if the terms neglected in the derivation of Eq.’s (18-21) are justified, further analyses are performed.

7.1 Simulations of the experiments

To further analyse the validity of the experiments, a FE-analysis is performed. The derived constitutive relation is used with the finite element method. Three conditions have to be fulfilled for the experimental method to be valid; the adherends must deform according to elastic Euler-Bernoulli beam theory, the stress-deformation relation has to be unique along the layer and the distance between the crack tip and the loading point, \( b \), cf. Fig. 5, has to be long enough. The one condition that can be checked in the FE-simulation is if the distance \( b \) is long enough. The conditions are:

(i) \( v(b) \ll v(0) \)

(ii) \( v'(b) \) should be possible to estimate using a linear elastic adhesive, cf. Eq. (29).

In these simulations the commercial program ABAQUS (v. 6.4) is used. Interface elements by Salomonsson (2002) are used to model the adhesive layer. The interface elements are further developed to include the softening part of the \( \tau-V_0 \)-curve from the experiments. The interface elements have three degrees of freedom at each node, cf. Fig. 37. Every interface element consists of four nodes, two attached to each adherend. The adherends are modelled with beam elements that deform according to the Timoshenko beam theory. The element type B21 is used.

![Figure 37](Image)

**Figure 37** The interface element used in the FE-analysis.
The constitutive relations from the experiments are inserted into the FE-model. For every experiment a $\tau-v_0$-curve is constructed with 12 points to approximate the constitutive relation, cf. Fig. 38.

Figure 38 The stress-deformation relation used in the FE-simulation is an approximation of the cohesive relation from the experiments (dotted line). The curve is approximated by twelve points with straight lines in between, i.e. the solid line in the graph. The present curve is from experiment J.

Figure 39 shows the simulated and experimental $P-\Delta$-curves for experiments J, L and N. The agreement is good between the experimental curve (solid line) and the simulated curve (dashed line). After the value of the maximum force is reached, the curves diverge slightly for the specimens J and L. The divergence is probably due to the fact that the value of the energy release rate is assumed to be constant and equal to the fracture energy, $J_c$, in the simulations. In the experiments the value of $J_{ad}$ decreases slightly after fracture, cf. Fig. 23. In experiment N, the value of the fracture energy does not decrease; hence, the good agreement between the simulation and experiment in the lower part of Fig. 39. The generally good agreement between the simulated and experimental data is an indication of the good quality of the method.
Figure 9 shows the shear stress distribution in the adhesive layer at the critical load. In Fig. 40, the distribution of shear deformation is shown for specimen N. As shown in the figure, \( v(b) \) is about 0.5 \( \mu \)m at fracture. This value should be compared to \( v(0) \) which is equal to 0.14 mm. Thus, it is safe to neglect \( v(b) \) as compared to \( v(0) \). The neglected term is: 

\[
3Pv(b)/8WH
\]

At fracture this term is less than 2 J/m^2.

A second term that is neglected by the same reasoning is

\[
\int_0^{v(b)} \tau(v)dv = \frac{1}{2}kv(b)^2
\]

(57)

The equality holds because the shear deformation of the adhesive layer is small under the loading point. The term contributes with 0.6 J/m^2. These two contributions enter with different signs, cf. Eq. (27). Thus, the contribution to \( J \) from the two terms above is 1.4 J/m^2 at the critical load for specimen N.

Figure 39 Force – deflection curves for specimens; J (upper left corner), L (upper right corner) and N (lower).
The second simplification is that the term $v'(b)$ can be approximated with Eq. (29) derived using a linear elastic model for the adhesive layer. This is also checked using FE-analyses. In the derivation of Eq. (23), it was shown that

$$\varepsilon_u - \varepsilon_{xx} = \varepsilon_{xx} - \varepsilon_{xx}(b),$$

where the last equality follows from the definition of strain. The strain components in the beam elements are given by the FE-analysis. This gives $v'(b) = 1.2 \cdot 10^{-4}$ as a typical result. In Eq. (30), the value of $v'(b)$ is approximated with $v'(b)$ from an analysis using an elastic adhesive, cf. Eq. (29). This value is $v'(b) = 0.7 \cdot 10^{-4}$ for specimen N. Though the error is substantial, this value contributes to the third term in Eq. (30), $J_2$, which is very small, cf. Fig. 10. Thus, the error is less than $3.6 \text{ J/m}^2$ using the elastic analysis. The total error due to these simplifications are summarised below. The size of the error is given by

$$E_j = -\tau v(b) + \int_0^{v(b)} \tau(v) \, dv - \left( \frac{EH}{16} v'(b)^2 - \frac{9}{128} \frac{P^2}{kW^2H^2} \right).$$

(58)

These simplifications give a negative error, cf. Fig 41, i.e. the fracture energy is slightly overestimated.

**Figure 40** The distribution of the shear deformation at fracture along the adhesive layer, from experiment N.
As expected, the error increases with the height of the specimen since the process zone is longer for a larger $H$, cf. Fig 28.

It is observed in Fig. 39 that the applied force first decreases and then increases after the crack has started to propagate. The reason for this is indicated in Fig. 42. When the crack has propagated about 110 mm, the process zone is situated at a position under the load application point. With further propagation of the crack, the process zone spreads to the right of the loading point and the load increases again. This phenomenon is observed both in the experiments and in the simulations.

**Figure 41** The error due to the simplifications $v(b) \ll v(0)$ and the estimate of $v'(b)$ from a linear elastic theory for the adhesive.

**Figure 42** The shear stress distribution along the adhesive layer when the crack has propagated 110 mm, from experiment J.
7.2 Neglected terms in the $J$-integral

When deriving the different terms of the energy release rate, Eq. (17), some simplifications were made. These will be explored here.

In the expressions for the contributions to the $J$-integral from the paths B and D in Eq.’s (18) and (19), two additional terms containing the traction vector were neglected. These terms contribute with

$$\int \left( \sigma_{xx} \frac{\partial u_x}{\partial x} + \sigma_{yx} \frac{\partial u_y}{\partial x} \right) dy. \tag{59}$$

where the paths B and D are taken within the adhesive layer and very close to the boundaries. Close to the crack tip, (B), $\sigma_{xx}$ is very small since the boundary is stress free. Thus, the first term in Eq. (59) does not contribute to $J$ for path B. At the loading point, (D), the strain $\varepsilon_{xx}=\partial u_y/\partial x$ is an odd function of $y$. The stress, $\sigma_{xx}$, is dominated by the constraints of the adherends and the large vertical stress, $\sigma_{yy}$, cf. Fig. 43. Thus, $\sigma_{xx}$ is almost constant over the thickness of the adhesive. The contribution to the $J$-integral from the first term in Eq. (59) is therefore small. The other term, $\sigma_{yx}$, gives the contribution:

$$\sigma_{yx} \frac{\partial u_y}{\partial x} \tag{60}$$

This term will be small since the contribution to the shear strain mainly originates from the term $\partial u_x/\partial y$. Thus,

$$\frac{\partial u_x}{\partial y} \gg \frac{\partial u_y}{\partial x} \tag{61}$$

In the derivation of Eq.’s (18) and (19), contributions to the strain energy are also neglected. These terms contribute with $\sigma_{xx} d\varepsilon_{xx}$ and $\sigma_{yy} d\varepsilon_{yy}$ to the strain energy term in Eq. (1). As described above, $\sigma_{xx} d\varepsilon_{xx}$ gives a small contribution at B and D. Moreover, at B, $\sigma_{yy}$ is almost zero, cf. Fig. 43. The contribution from the term $\sigma_{yy} d\varepsilon_{yy}$, at D, is $1\sigma_{yy} d\varepsilon_{yy}/2$. This term contributes less than 1 % to the total value of the fracture energy. Thus, the contribution to Eq.’s (18) and (19) is very small.
In the expressions for the contributions to the $J$-integral from the paths A and C, cf. Eq. (20) and (21), an additional term originating from the normal stress $\sigma_{yy}$ is neglected. Along the adhesive layer this normal stress is very small, except at the loading point where a compressive stress appears, cf. Fig. 43.

![Figure 43](image-url)  

**Figure 43**  
The stress distribution along the adhesive layer at the start of crack propagation for specimen N. Normal stress (dotted line) and shear stress (solid line).

At the loading point this stress will be much larger than the shear stress. The contribution to the $J$-integral is

$$
\int_{0}^{b} \sigma_{yy} \frac{\partial u_{y}}{\partial x} \, dx.
$$

(62)

The term $\frac{\partial u_{y}}{\partial x}$ is identified as the rotation of the specimen. Since the specimen is almost symmetric with respect to the loading point, the rotation is very small at the loading point where the stress is large. Moreover, although the normal stress is large under the loading point, the other term, containing the shear stress, contributes with a large shear stress over most of the integration path. Thus, the contribution from Eq. (62) is small.
The experimental method described in this paper enables the derivation of the complete stress-deformation relations for thin adhesive layers loaded in shear. The method is based on an inverse analysis. Experiments on three different geometries of the specimen are performed. It is shown that the stress-deformation relation does not depend on the dimensions of the adherends. Thus, the constitutive relation can be considered to be a property of the adhesive layer. Two different experimental conditions are used; one with constant rate of displacement of the loading point and one with constant shear deformation rate. For the experiments with constant deflection rate, the average value of the fracture energy and the shear deformation at fracture are 3.18 kJ/m$^2$ and 0.18 mm, respectively. For the experiments with a constant shear deformation rate, the values of the fracture energy and shear deformation at fracture are 2.27 kJ/m$^2$ and 0.16 mm, respectively. The difference in fracture energy is probably due to the different shear deformation rates. In all experiments the fracture mode is cohesive. Further investigations are necessary to determine the dependence of the stress-deformation relation on the thickness of the adhesive layer, shear deformation rate, type of adhesive etc. The results show no specific dependence on the batches of the adhesive.

Strength analysis of adhesively joined structures based on the adhesive layer theory requires that the microscopic variations of the stress and strain fields within the adhesive layer have a negligible influence on the stress-deformation relation for the adhesive layer. We have showed that the shear stress-shear deformation relation for an engineering adhesive (DOW Betamate XW1044-3) is unaffected by variations in the size of the process zone. However, the process zone is large in all the experiments reported here. A method of achieving a shorter process zone is to allow the adherends to deform plastically. The validity of the $\tau$-$\gamma_0$-relation needs to be controlled for these cases.

Monte-Carlo simulations show that the method is not sensitive to randomised error during the experiment. Moreover, systematic errors in the measurement system and errors in dimensional data and Young’s modulus of the adherends result in small errors in the constitutive relation.
REFERENCES


APPENDIX:

Calibration curves